Proceedings of the Seventh IASTED International Conference
POWER AND ENERGY SYSTEMS
November 28 - December 1, 2004, Clearwater Beach, FL, USA

INTERNAL FAULTS IN SYNCHRONOUS MACHINES: NETWORK MODEL FOR INTERNAL FAULT ANALYSIS

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ABSTRACT

This paper treats a network model for calculating short-circuit currents from internal shunt short-circuits in synchronous machines, and describes an operationally dual method to the method previously employed to analyze internal shunt faults in synchronous machines. The model for internal shunt short-circuits is derived by theoretical means under the assumption that the machine considered is an ideal synchronous machine, by initially neglecting all resistances in the machine. Next, internal series faults in synchronous machines are dealt with by applying the organized cut-set method by Gabriel Kron. One or several simultaneous faults can be treated by the same matrix equation. The network model is to be used in conjunction with a model for the internal impedances in faulted machines. On the basis that the machine acts as an ideal synchronous machine, it can be represented in symmetrical components. The model is analytically derived using only those data that manufactures normally provide. The calculations are done, assuming that the respective internal impedances can be calculated, by means of applying a relatively simple network model for the faulted machine. This approach uses Thevenins principle for calculating short-circuits.

KEY WORDS

Synchronous machines, machine network model, internal faults, Thevenins principle, Kronian organized mesh method.

1 INTRODUCTION

Internal faults in synchronous machines are in general rare, but they occur occasionally due to e.g. insulation failures. These faults may be categorized as either shunt short-circuits or series faults. These differ somewhat in their nature, but both cause the fault by altering the internal impedances of the machine. In this paper, a network model for analyzing shunt short-circuits and series faults are treated.

Previous studies have been carried out within this area [1-6, 25]. However, most of these studies use a rather large amount of data and complicated mathematics, such as e.g. FEM. This makes the model more complex and harder to understand the physical system, hereby, making the interpretation of the model results is made more difficult.

The model presented in this paper is derived using the representation of the ideal synchronous machine [12] in symmetrical components, see [20]. This reduces the amount of needed data to "nameplate-data" for the machine alone.

A series fault may occur as a complete or partial fracture of parts of the phase-conductors or other situations where the admittance of a conductor is altered from what was intended. In the model presented in this paper, it is assumed that there is no initial shunt short-circuit to either earth or other conducting parts at different potentials. Such situations may occur where conducting parts are poorly shunted, forming an asymmetry in the system. Initially, such an asymmetrical altering of the internal admittances of the machine will result in a voltage drop over the fault part of the conductor, which may be interpreted as the fault voltage, and which may even lead to a further enlargement of the fault.

2 BASIC MACHINE AND SHUNT SHORT-CIRCUIT CALCULATION PROPERTIES

An ideal synchronous machine can normally be represented by three states, describing the synchronous machine under different conditions [21]:

- Sub-transient state, where the machine is operating under extensively disturbed conditions, for which no magnetic stability is present (e.g. during short-circuits)
- Transient state, where the machine is operating under disturbed conditions, moving towards a magnetic stability
- Steady state, where the machine is operating under normal, steady conditions, magnetically stable

When a machine is undergoing internal short-circuits it must be presumed that the sub-transient description of the machine is the most appropriate.

From basic synchronous machine theory [20] an ideal synchronous machine in its sub-transient state and connected to an adjacent network can be represented in symmetrical components as shown in Fig. 1 [24].

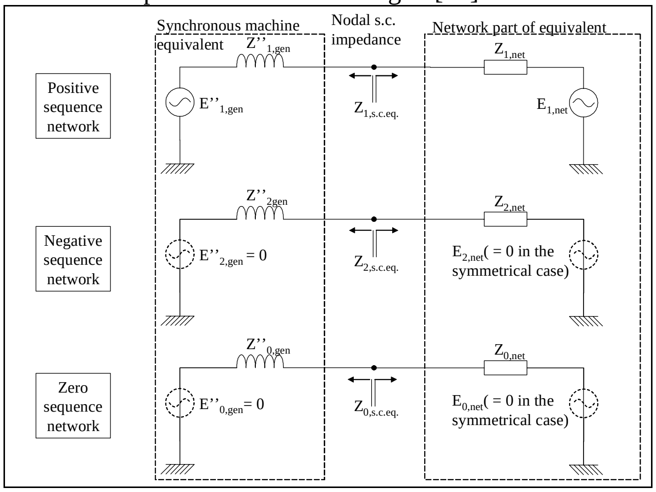

Fig. 1. Symmetrical component short-circuit equivalent for synchronous machine connected to an adjacent network

The impedances in this symmetrical representation can, according to [20, 24], be expressed (when neglecting resistances) as

$$Z''_{1,gen} \approx j \cdot X''_d \tag{1a}$$

$$Z''_{2,gen} \approx j \cdot X_2 = j \cdot \left(\sqrt{\left(X''_d + \frac{X_0}{2} \right)\left(X''_q + \frac{X_0}{2} \right)} - \frac{X_0}{2} \right) \tag{1b}$$

$$Z''_{0,gen} \approx Z_{g,0} + j \cdot X_0 \tag{1c}$$

where X''_d is the direct-axis reactance of the synchronous machine, X''_q is the quadrature-axis reactance of the synchronous machine, X_0 is the zero-sequence reactance of the machine, and $Z_{g,0}$ is the synchronous machine earthing impedance between the machine neutral and earth. $Z''_{1,gen}$ is the synchronous machine positive sequence impedance, and $Z''_{2,gen}$ and $Z''_{0,gen}$ are the negative and zero sequence impedance of the synchronous machine, respectively. Behind the machine impedances are the respective synchronous machine emfs, expressed in terms of symmetrical components. In Fig.1 the adjacent network is also represented by its symmetrical representation.

If the fault current, $\bar{I}_{f,sym}$, of any arbitrary shunt fault on the terminals of the machine, connected to the adjacent network, is desired, then it can be calculated using Gabriel Kron's organized mesh method of [23] as done in [21]. This gives

$$\begin{aligned} \bar{I}_{f,sym} = {} & \bar{\bar{S}}^{-1} \times \bar{\bar{C}}_T \\ & \times \left(\bar{\bar{C}}_T^{t} \times \bar{\bar{S}} \times \bar{\bar{Z}}_{s.c.eq,sym} \times \bar{\bar{S}}^{-1} \times \bar{\bar{C}}_T + \bar{\bar{Z}}_f \right)^{-1} \\ & \times \bar{\bar{C}}_T^{t} \times \bar{\bar{S}} \times \bar{u}_{th.eq.,sym} \end{aligned} \tag{2}$$

where $\bar{\bar{S}}$ is the mathematical Fortescue transformation matrix between symmetrical components time independent phase-values, $\bar{\bar{Z}}_{s.c.eq,sym}$ is a diagonal system impedance matrix, containing the nodal impedances of the short-circuit equivalent, $\bar{\bar{C}}_T$ is the tree-part of the mesh-matrix, describing the topology of the applied short-circuit [21, 23], and $\bar{\bar{Z}}_f$ is the impedance matrix, containing the impedances of the applied short-circuit and the vector $\bar{u}_{th.eq.,sym}$ contains the pre-fault Thevenin emfs in symmetrical components in the fault location.

The Fortescue transformation matrix is defined as

$$\bar{\bar{S}} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \tag{3}$$

with

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j\frac{2\pi}{3}} \tag{4}$$

In this formulation of the Fortescue matrix, the order of the sequences is positive sequence (1-sequence), negative sequence (2-sequence) and zero-sequence (0-sequence).

The system impedance matrix is defined as

$$\bar{\bar{Z}}_{s.c.eq,sym} = \begin{bmatrix} Z_{1,s.c.eq.} & 0 & 0 \\ 0 & Z_{2,s.c.eq.} & 0 \\ 0 & 0 & Z_{0,s.c.eq.} \end{bmatrix} \tag{5a}$$

yielding

$$\bar{\bar{Z}}_{s.c.eq,sym} = \begin{bmatrix} \left(\frac{1}{Z''_{1,gen}} + \frac{1}{Z_{1,net}} \right)^{-1} & 0 & 0 \\ 0 & \left(\frac{1}{Z''_{2,gen}} + \frac{1}{Z_{2,net}} \right)^{-1} & 0 \\ 0 & 0 & \left(\frac{1}{Z''_{0,gen}} + \frac{1}{Z_{0,net}} \right)^{-1} \end{bmatrix} \tag{5b}$$

The voltage vector $\bar{u}_{th.eq.,sym}$ can usually be set to be

$$\bar{u}_{th.eq.,sym} = \begin{bmatrix} u_{1,nom} \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

where $u_{1,nom}$ is the nominal positive sequence voltage of the network.

The expression for the resulting initial short-circuit current's a.c.-component in (2) can be used for any arbitrary configuration of fault, making the method quite general for fault calculation

For a single phase to ground short circuit in phase a, the mesh matrix becomes

$$\overline{\overline{C}}_T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

The corresponding fault current is then given by

$$I_f = \frac{3 \cdot u_{1,nom}}{\sqrt{3} \cdot (Z''_{1,gen} + Z''_{2,gen} + Z''_{0,gen} + 3 \cdot Z_f)} \quad (8a)$$

or

$$I_f \approx \frac{\sqrt{3} \cdot u_{1,nom}}{X''_d + X_2 + X_0 + Z_{g,0} + 3 \cdot Z_f} \quad (8b)$$

which can be found in the literature, e.g. [20].

3 THEORETICAL PHASE-VALUE MODEL FOR A SYNCHRONOUS MACHINE

From the symmetrical representation of the synchronous machine, an impedance matrix, containing the impedances of the machine, can be set up as

$$\overline{\overline{Z}}''_{synchsym.} = \begin{bmatrix} Z''_{1,gen} & 0 & 0 \\ 0 & Z''_{2,gen} & 0 \\ 0 & 0 & Z''_{0,gen} - Z_{g,0} \end{bmatrix} \quad (9a)$$

or

$$\overline{\overline{Z}}''_{synchsym.} \approx j \cdot \begin{bmatrix} X''_d & 0 & 0 \\ 0 & X_2 & 0 \\ 0 & 0 & X_0 \end{bmatrix} \quad (9b)$$

separating the grounding impedance of the machine neutral from the machine description.

If these values are transformed from symmetrical components to time independent phase values, simply by the use of the Fortescue matrix $\overline{\overline{S}}$, the machine impedance matrix becomes

$$\overline{\overline{Z}}''_{synch.ph} = \overline{\overline{S}} \times \overline{\overline{Z}}''_{synch.sym} \times \overline{\overline{S}}^{-1} \quad (10a)$$

or

$$\overline{\overline{Z}}''_{synch.ph} = \begin{bmatrix} Z''_e & Z''_g & \zeta''_g \\ \zeta''_g & Z''_e & Z''_g \\ Z''_g & \zeta''_g & Z''_e \end{bmatrix} \quad (10b)$$

where Z''_e is the self-impedance of each of the phase-winding equivalents in the synchronous machine representation, and Z''_g and ζ''_g are the mutual impedances of the synchronous machine.

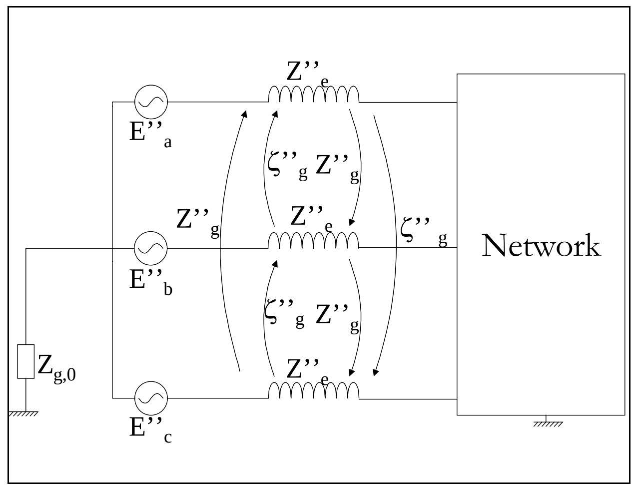

Fig. 2. Theoretical phase model of synchronous machine.

These derived theoretical phase-values can physically be interpreted as the self- and mutual impedances in Fig. 2 and can be calculated as

$$Z''_e = \frac{Z''_{1,gen} + Z''_{2,gen} + Z''_{0,gen}}{3} \approx j \cdot \frac{X''_d + X_2 + X_0}{3}, \quad (11)$$

$$Z''_g = \left(\frac{Z''_{0,gen}}{3} - \frac{Z''_{1,gen} + Z''_{2,gen}}{6} \right) + j \cdot \left(\frac{\sqrt{3}(Z''_{1,gen} - Z''_{2,gen})}{6} \right) \quad (12a)$$

or

$$Z''_g \approx -\frac{\sqrt{3}(X''_d - X_2)}{6} - j \cdot \frac{X''_d + X_2 - 2X_0}{6}, \quad (12b)$$

and

$$\zeta''_g = \left(\frac{Z''_{0.gen}}{3} - \frac{Z''_{1.gen} + Z''_{2.gen}}{6} \right) - j \cdot \left(\frac{\sqrt{3}(Z''_{1.gen} - Z''_{2.gen})}{6} \right) \quad (13a)$$

or

$$\zeta''_g \approx +\frac{\sqrt{3}\left(X''_d - X_2\right)}{6} - j \cdot \frac{X''_d + X_2 - 2X_0}{6} \tag{13b}$$

in terms of the symmetrical components of the synchronous machine.

Equally, the emfs of the machine can be transformed to theoretical phase values, by use of the Fortescue matrix. This leads to

$$\overline{E}''_{synch.ph} = \overline{\overline{S}} \times \overline{E}''_{synch.sym} = \overline{\overline{S}} \times \begin{bmatrix} E''_d \\ 0 \\ 0 \end{bmatrix} \tag{14a}$$

$$\overline{E}''_{synch.ph} = \begin{bmatrix} E''_d \\ a^2 \cdot E''_d \\ a \cdot E''_d \end{bmatrix} = \begin{bmatrix} E''_a \\ E''_b \\ E''_c \end{bmatrix} \tag{14b}$$

where E''_d is the direct-axis sub-transient emf of the machine

4 THE PARTITIONED PHASE-VALUE MACHINE MODEL

Considering a machine in its pre-fault state, but regarding the machine from a position inside it, the description becomes somewhat different from a machine description from the terminals.

Seen from inside, a model of a synchronous machine can be set up, using the philosophy shown in Fig. 3. If a machine is observed from an internal position, characterized by the arbitrarily chosen relative fault location at the positions α, β and γ, the machine impedances split up in three kinds of impedances:

Phase-winding self-impedances (e.g. Z''_{aas} or Z''_{aat}) related to the star-point (neutral) region and the terminal region, forming the self-impedances of the respective phase-winding partitions.

Intra-phase mutual impedances (e.g. Z''_{asat}), that are a consequence of the mutual impedance in-between the phase-winding partitions

Inter-phase mutual impedances (e.g. Z''_{asbt} or ζ''_{asbt}), that are a consequence of the sharing of flux in-between phases-winding partitions of different phases

To represent the machine in a satisfactory way, all these impedances must be determined.

As commonly known, the adjacent network can be described in symmetrical components by an impedance matrix

$$\overline{\overline{Z}}_{net,sym} = \begin{bmatrix} Z_{1,net} & 0 & 0 \\ 0 & Z_{2,net} & 0 \\ 0 & 0 & Z_{0,net} \end{bmatrix} \tag{15}$$

which contains an equal description in phase-value components given as

$$\begin{aligned} \overline{\overline{Z}}_{net.ph} &= \overline{\overline{S}} \times \overline{\overline{Z}}_{net.sym} \times \overline{\overline{S}}^{-1} \\ &= \begin{bmatrix} Z_{e,net} & Z_{g,net} & Z_{g,net} \\ Z_{g,net} & Z_{e,net} & Z_{g,net} \\ Z_{g,net} & Z_{g,net} & Z_{e,net} \end{bmatrix} \end{aligned} \tag{16}$$

Adapting the model from Fig. 3, an impedance matrix for the total system with reference in the star-point, seen from an internal location in the relative distances α, β and γ between the machine neutral and terminals, becomes

$$\overline{\overline{Z}}''_{system.ph.} = \begin{bmatrix} Z''_{g,0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z''_{aa,s} & Z''_{asbs} & \zeta''_{ascs} & Z''_{asat} & Z''_{asbt} & \zeta''_{asct} \\ 0 & \zeta''_{asbs} & Z''_{bb,s} & Z''_{bscs} & \zeta''_{bsat} & Z''_{bsbt} & Z''_{bsct} \\ 0 & Z''_{ascs} & \zeta''_{bscs} & Z''_{cc,s} & Z''_{csat} & \zeta''_{csbt} & Z''_{csct} \\ 0 & Z''_{asat} & Z''_{bsat} & \zeta''_{csat} & Z''_{aa,t}+Z_{e,net} & Z''_{atbt}+Z_{g,net} & \zeta''_{atct}+Z_{g,net} \\ 0 & \zeta''_{asbt} & Z''_{bsbt} & Z''_{csbt} & \zeta''_{atbt}+Z_{g,net} & Z''_{bb,t}+Z_{e,net} & Z''_{btct}+Z_{g,net} \\ 0 & Z''_{asct} & \zeta''_{bsct} & Z''_{csct} & Z''_{atct}+Z_{g,net} & \zeta''_{btct}+Z_{g,net} & Z''_{cc,t}+Z_{e,net} \end{bmatrix} \tag{17}$$

where the parameters refer to the notation of Fig. 3.

Assuming that all data are known, the corresponding nodal impedance matrix can be calculated, by firstly stating that

$$\overline{\overline{Y}}''_{system.ph.} = \left(\overline{\overline{Z}}''_{system.ph}\right)^{-1} \tag{18}$$

From a graph theoretical analysis of the network in Fig. 3, the nodal impedance matrix of the total network becomes

$$\left(\overline{\overline{A}}^t \times \overline{\overline{Y}}''_{sys.ph} \times \overline{\overline{A}}\right)^{-1} = \begin{bmatrix} Z''_S & Z''_{S\alpha} & Z''_{S\beta} & Z''_{S\gamma} \\ Z''_{\alpha S} & Z''_{\alpha} & Z''_{\alpha\beta} & \zeta''_{\alpha\gamma} \\ Z''_{\beta S} & \zeta''_{\alpha\beta} & Z''_{\beta} & Z''_{\beta\gamma} \\ Z''_{\gamma S} & Z''_{\alpha\gamma} & \zeta''_{\beta\gamma} & Z''_{\gamma} \end{bmatrix} \tag{19}$$

where $\overline{\overline{A}}$ is the nodal incidence matrix.

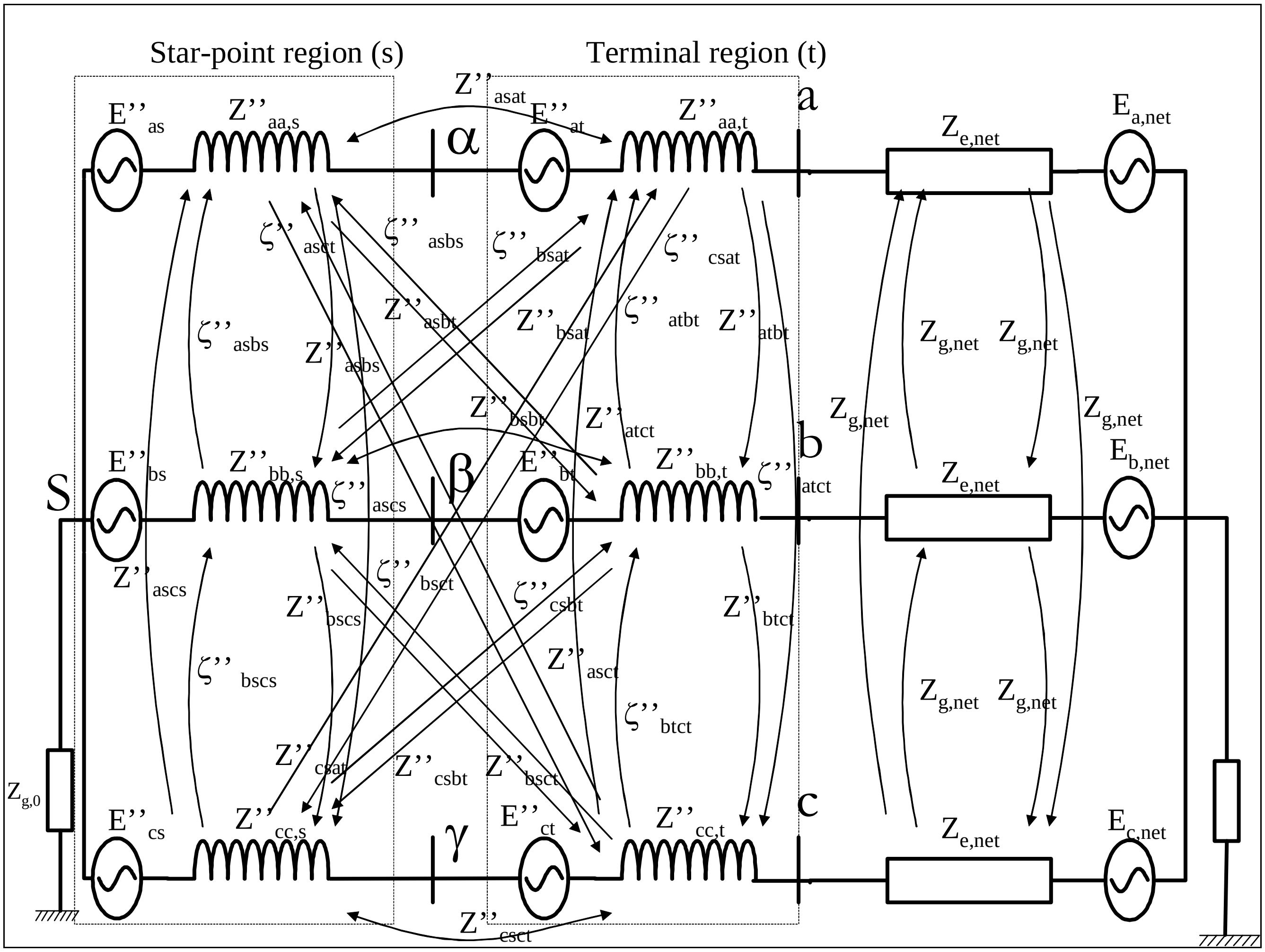

Fig. 3. Theoretical phase-value representation of a partitioned synchronous machine connected to an adjacent network; the adjacent network is represented by its symmetrical components, and the machine neutral (S) is earthed through an impedance, $Z_{g,0}$. In this figure, the suffixes of the impedances relate to the coupling that these describe, so that e.g. the impedance Z''_{asbt} is the mutual coupling from the star-point region of phase a to the terminal region of phase b.

As only the impedance relations for the nodes located at α, β and γ are of interest in a calculation of the short-circuit current from a shunt short-circuit in this position, the relevant parts of this matrix can be cut out of the matrix in (19), forming the matrix

$$\overline{\overline{Z}}''_{nodalsys.ph} = \begin{bmatrix} Z''_{\alpha} & Z''_{\alpha\beta} & \zeta''_{\alpha\gamma} \\ \zeta''_{\alpha\beta} & Z''_{\beta} & Z''_{\beta\gamma} \\ Z''_{\alpha\gamma} & \zeta''_{\beta\gamma} & Z''_{\gamma} \end{bmatrix} \tag{20}$$

which in symmetrical components can be expressed as

$$\begin{aligned} \overline{\overline{Z}}''_{nodalsys.sym} &= \overline{\overline{S}}^{-1} \times \overline{\overline{Z}}''_{nodalsys.ph} \times \overline{\overline{S}} \\ &= \begin{bmatrix} Z''_{P1} & Z''_{P1P2} & \zeta''_{P1P0} \\ \zeta''_{P1P2} & Z''_{P2} & Z''_{P2P0} \\ Z''_{P1P0} & \zeta''_{P2P0} & Z''_{P0} \end{bmatrix} \end{aligned} \tag{21}$$

From the method of calculating short-circuit currents given in (2), the resulting short-circuit current can be calculated as

$$\begin{aligned} \overline{I}_{f,sym} &= \overline{\overline{S}}^{-1} \times \overline{\overline{C}}_T \times \\ &\left(\overline{\overline{C}}_T^{\,t} \times \overline{\overline{S}} \times \overline{\overline{Z}}''_{nodalsys.sym} \times \overline{\overline{S}}^{-1} \times \overline{\overline{C}}_T + \overline{\overline{Z}}_f \right)^{-1} \times \\ &\overline{\overline{C}}_T^{\,t} \times \overline{\overline{S}} \times \overline{E}''_{th.eq.,sym} \end{aligned} \tag{22a}$$

i.e.

$$\begin{aligned} \overline{I}_{f,sym} &= \overline{\overline{S}}^{-1} \times \overline{\overline{C}}_T \times \\ &\left(\overline{\overline{C}}_T^{\,t} \times \overline{\overline{S}} \times \begin{bmatrix} Z''_{P1} & 0 & 0 \\ 0 & Z''_{P2} & 0 \\ 0 & 0 & Z''_{P0} \end{bmatrix} \times \overline{\overline{S}}^{-1} \times \overline{\overline{C}}_T + \overline{\overline{Z}}_f \right)^{-1} \times \\ &\overline{\overline{C}}_T^{\,t} \times \overline{\overline{S}} \times \begin{bmatrix} E''_{P1} \\ E''_{P2} \\ E''_{P0} \end{bmatrix} \end{aligned} \tag{22b}$$

where the emfs in (22b) are given as

$$\overline{E}''_{th.eq.,sym} = \begin{bmatrix} E''_{P1} \\ E''_{P2} \\ E''_{P0} \end{bmatrix} = \overline{\overline{S}}^{-1} \times \begin{bmatrix} E''_{\alpha} \\ E''_{\beta} \\ E''_{\gamma} \end{bmatrix} \quad (23)$$

where E''_{α}, E''_{β} and E''_{γ} are the internal phase-value Thevenin emfs at the respectively positions α, β and γ inside the machine. The expression in (22) can, as (2), handle any arbitrary configuration of the short-circuit applied, which expands to application of the method.

As stated by Kron [23] the tree is chosen as the system seen from the faulted positions. Hence, each connection in the fault defines a mesh, described by the mesh matrix $\overline{\overline{C}}$. The tree-part of the mesh-matrix, $\overline{\overline{C}}_T$, is given in accordance with the configuration of the fault apparatus, yielding for a single-phase short-circuit (in phase a), a two-phase short-circuit without earth (in phase a and b) or a three-phase short-circuit to earth

$$\overline{\overline{C}}_T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{(24)}$$

$$\overline{\overline{C}}_T = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad (25)$$

or

$$\overline{\overline{C}}_T = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ -1 & 0 \end{bmatrix} \quad (26)$$

respectively.

Other possible matrix-forms may be applied, according to [23] and/or [21].

5 BOUNDARY CONDITIONS FOR INTERNAL IMPEDANCES

In order to be able to apply the above given method for calculation, it is crucial that proper modeled impedances are applied in the model. Thus the scale of the self- and mutual impedances are very dependent on the location of the fault, and hereby on the relative positions α, β and γ inside the machine, where the fault occurs.

Considering the values of the self- and mutual impedances in same phases (e.g. $Z''_{aa,s}$, $Z''_{aa,t}$ and Z''_{asat} for phase a) these form what originally would form the self impedance, Z''_e, of the respective phase-windings, yielding, as impedances are additive in behavior with respect to the theoretical phase-model in Fig. 3 that

$$Z''_e = Z''_{aa,s} + Z''_{aa,t} + 2 \cdot Z''_{asat} \quad (27a)$$

or

$$Z''_e = Z''_{bb,s} + Z''_{bb,t} + 2 \cdot Z''_{bsbt} \quad (27b)$$

or

$$Z''_e = Z''_{cc,s} + Z''_{cc,t} + 2 \cdot Z''_{csct} \quad (27c)$$

which form a boundary condition for the consisting impedances. Adapting this point of view to the mutual couplings, a similar set of boundary conditions can be set up for the mutual impedances, yielding

$$Z''_g = Z''_{asbs} + Z''_{asbt} + Z''_{atbs} + Z''_{atbt} \quad (28a)$$

or

$$Z''_g = Z''_{ascs} + Z''_{asct} + Z''_{atcs} + Z''_{atct} \quad (28b)$$

or

$$Z''_g = Z''_{bscs} + Z''_{bsct} + Z''_{btcs} + Z''_{btct} \quad (28c)$$

and

$$\zeta''_g = \zeta''_{asbs} + \zeta''_{asbt} + \zeta''_{atbs} + \zeta''_{atbt} \quad (29a)$$

or

$$\zeta''_g = \zeta''_{ascs} + \zeta''_{asct} + \zeta''_{atcs} + \zeta''_{atct} \quad (29b)$$

or

$$\zeta''_g = \zeta''_{bscs} + \zeta''_{bsct} + \zeta''_{btcs} + \zeta''_{btct} \quad (29c)$$

forming the boundary condition of the mutual couplings.

For the emfs an equal boundary condition can be set up, requiring that the emfs (regarded as vectors) fulfill

$$E''_a = E''_{as} + E''_{at}, \quad (30a)$$

$$E''_b = E''_{bs} + E''_{bt} \quad (30b)$$

and

$$E''_c = E''_{cs} + E''_{ct} \quad (30c)$$

It cannot be excluded that there may be many ways to model the internal impedances of a synchronous machine, with respect to the internal geometric configuration of the regarded machine, but however the modeling is carried out, it must fulfill these boundary conditions.

6 THE DUALITY BETWEEN SHUNT AND SERIES FAULTS

As shown in sections 2 to 5, shunt short-circuits can be evaluated by means of mesh calculations using Thevenin equivalents. Thus, when calculating what in network theory [21, 24] is dual to calculating shunt short-circuit, namely series faults, it can be done using the operationally dual method, i.e. cut-set methods, introducing the organized Kronian cut-set method [21, 24]. According to network theory [24], the dual network to the Thevenin equivalent is the Norton equivalent.

When calculating shunt short-circuits, the applied Thevenin theorem in the short-circuit current calculation says, that the Thevenin voltage in the fault should be identical to the pre-fault voltage. Analogously, when considering series faults, and therefore Norton equivalents, the analogous theorem must yield that the fault voltage must be calculated on basis of a Norton current identical to the pre-fault current through the subjected conductor.

Hence, in the case of series fault calculation, the pre-fault current must be known to calculate the resulting change in voltage. By using the organized cut-set method for calculating network voltages on the basis of given currents [24], the resulting matrix expression for the fault voltage, $\bar{u}_{f,sym}$, in symmetrical components becomes [21]

$$\begin{aligned}\bar{u}_{f,sym} &= \bar{\bar{S}}^{-1} \times \bar{\bar{D}}_L \\ &\times \left(\bar{\bar{D}}_L^{t} \times \bar{\bar{S}} \times \bar{\bar{Y}}_{s.c.eq,sym} \times \bar{\bar{S}}^{-1} \times \bar{\bar{D}}_L + \bar{\bar{Y}}_f \right)^{-1} \\ &\times \bar{\bar{D}}_L^{t} \times \bar{\bar{S}} \times \bar{i}_{no.eq.,sym}\end{aligned} \tag{31}$$

where $\bar{\bar{S}}$ is the mathematical Fortescue transformation matrix between the symmetrical components time indepentent phase-values, $\bar{\bar{Y}}_{s.c.eq,sym}$ a diagonal admittance matrix containing the mesh admittances of the fault equivalent, $\bar{\bar{D}}_L$ is the link-part (i.e. co-tree part) of the cut-set matrix [24], describing the configuration of the fault [3], and $\bar{i}_{no.eq.,sym}$ contains the pre-fault current of the machine in symmetrical components.

The admittance matrix is defined as

$$\bar{\bar{Y}}_{s.c.eq,sym} = \begin{bmatrix} Y_{1,s.c.eq.} & 0 & 0 \\ 0 & Y_{2,s.c.eq.} & 0 \\ 0 & 0 & Y_{0,s.c.eq.} \end{bmatrix} \tag{32}$$

The expression in (31) for the resulting fault overvoltages is dual to the expresson in (2) from which the short-circuit current could be calculated when a short-circuit occurs.

7 DETERMINING FAULT EQUIVALENT PROPERTIES

To determine the values of the mesh admittances of the fault equivalent, the mesh-properties of the fault equivalent network must be determined. As the equivalent network, describing the serial fault, is the same as the one describing the shunt short-circuits in sections 2 to 5, the analysis originates from the same network, keeping the same impedances, viz.

$$\bar{\bar{Z}}''_{system.ph.} = \begin{bmatrix} Z''_{g,0} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z''_{aa,s} & Z''_{asbs} & \zeta''_{ascs} & Z''_{asat} & Z''_{asbt} & \zeta''_{asct} \\ 0 & \zeta''_{asbs} & Z''_{bb,s} & Z''_{bscs} & \zeta''_{bsat} & Z''_{bsbt} & Z''_{bsct} \\ 0 & Z''_{ascs} & \zeta''_{bscs} & Z''_{cc,s} & Z''_{csat} & \zeta''_{csbt} & Z''_{csct} \\ 0 & Z''_{asat} & Z''_{bsat} & \zeta''_{csat} & Z''_{aa,t}+Z_{e,net} & Z''_{atbt}+Z_{g,net} & \zeta''_{atct}+Z_{g,net} \\ 0 & \zeta''_{asbt} & Z''_{bsbt} & Z''_{csbt} & \zeta''_{atbt}+Z_{g,net} & Z''_{bb,t}+Z_{e,net} & Z''_{btct}+Z_{g,net} \\ 0 & Z''_{asct} & \zeta''_{bsct} & Z''_{csct} & Z''_{atct}+Z_{g,net} & \zeta''_{btct}+Z_{g,net} & Z''_{cc,t}+Z_{e,net} \end{bmatrix} \tag{33}$$

For these impedances the boundary conditions given in section 5 apply.

From the network presented in Fig. 3, the mesh matrix as described by Kron in [21] and [24], becomes

$$\bar{\bar{C}} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{34}$$

as the system before the fault occurred is now perceived as meshes. Thus, the mesh admittance matrix in phase values becomes

$$\bar{\bar{Y}}''_{meshsys.ph} = \left(\bar{\bar{C}}^{t} \times \bar{\bar{Z}}''_{sys.ph} \times \bar{\bar{C}} \right)^{-1} \tag{35a}$$

or

$$\bar{\bar{Y}}''_{meshsys.ph} = \begin{bmatrix} Y''_{\alpha} & Y''_{\alpha\beta} & \psi''_{\alpha\gamma} \\ \psi''_{\alpha\beta} & Y''_{\beta} & Y''_{\beta\gamma} \\ Y''_{\alpha\gamma} & \psi''_{\beta\gamma} & Y''_{\gamma} \end{bmatrix} \tag{35b}$$

which when expressed in symmetrical components becomes

$$\overline{\overline{Y}}''_{meshsys.sym} = \overline{\overline{S}}^{-1} \times \overline{\overline{Y}}''_{meshsys.ph} \times \overline{\overline{S}} \quad (36a)$$

or

$$\overline{\overline{Y}}''_{meshsys.sym} = \begin{bmatrix} Y''_{M1} & Y''_{M1M2} & \psi''_{M1M0} \\ \psi''_{P1P2} & Y''_{M2} & Y''_{M2M0} \\ Y''_{M1M0} & \psi''_{P2P0} & Y''_{M0} \end{bmatrix} \quad (36b)$$

Due to Kirchhoff's current law, the pre-fault current through the faulted location, expressed in phase-values, is the same as the pre-fault currents through each phase, yielding that

$$\bar{i}_{no.eq.,sym} = \begin{bmatrix} i''_{M1} \\ i''_{M2} \\ i''_{M0} \end{bmatrix} \quad (37a)$$

or

$$\bar{i}_{no.eq.,sym} = \overline{\overline{S}}^{-1} \times \begin{bmatrix} i''_{a} \\ i''_{b} \\ i''_{c} \end{bmatrix} \quad (37b)$$

8 BUILDING THE CUT-SET MATRIX

To be able to build up the total expression, it is necessary to determine the link-part of the cut-set matrix, $\overline{\overline{D}}_L$, for the fault configuration. As described in both [24] and [21], a general method for setting up a link part cut-set matrix for a network is build by analyzing the network in the following way:

1. Choose an expanding tree for the network (and hereby indirectly also a set of links (co-tree))
2. Set up a matrix, holding the tree-branches of the network horizontally and the link branches vertically
3. For each of the branches of the tree, imagine a cut, causing the network to fall into two distinct parts, by cutting the regarded tree-branch and a number of links.
4. In the columns of each tree-branch insert for each link-row a
    a. 0 if the link does not have to be cut
    b. +1 if the link cut holds the same direction as the tree-branch regarded
    c. –1 if the link cut holds the opposite direction as the tree-branch regarded

For a system containing a machine and a network this should be represented as three meshes, one for each phase, shunted with any respective mutual admittances. For each distinct phase mesh, the pre-fault admittance is determined as the nodal admittance between reference and the fault location.

As it can hardly be imagined that series faults involve very complexly configured fault apparatuses, the usual configurations will be single-phased series faults, where the link part cut-set matrix for e.g. a fault in phase a is given as

$$\overline{\overline{D}}_L = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad (38)$$

a doubled phase serial fault for e.g. phases a and b, yielding

$$\overline{\overline{D}}_L = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad (39)$$

or the highly unlikely three-phase serial fault, yielding

$$\overline{\overline{D}}_L = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (40)$$

Now, knowing all parameters, the formula given in (31) can be used for calculation

9 A SMALL EXAMPLE: THE SINGLE PHASE SERIES FAULT

To stress the fact that there is a duality between the treatment of shunt short-circuits and series faults, the outcome of a series fault in a single phase is calculated.

Imagine a synchronous machine, in which phase a is suddenly subjected to a series fault.

Say that the seriesal fault has an admittance of Y_f, and that the synchronous machine has a symmetrical load, i.e. that

$$\bar{i}_{no.eq.,sym} = \overline{\overline{S}}^{-1} \times \begin{bmatrix} i''_{a} \\ i''_{b} \\ i''_{c} \end{bmatrix} = \begin{bmatrix} i''_{a} \\ 0 \\ 0 \end{bmatrix} \quad (41)$$

when $I''_{ph} = I''_a = I''_b = I''_c$

Supposing that the machine may be represented by its admittance representation in symmetrical components, i.e. as in (32), then the resulting initial fault voltage becomes (when using the cut-set matrix presented in (38))

$$\bar{u}_{f,sym} = \bar{\bar{S}}^{-1} \times \bar{\bar{D}}_L \times \left(\bar{\bar{D}}_L^t \times \bar{\bar{S}} \times \bar{\bar{Y}}_{s.c.eq,sym} \times \bar{\bar{S}}^{-1} \times \bar{\bar{D}}_L + \bar{\bar{Y}}_f \right)^{-1} \times \bar{\bar{D}}_L^t \times \bar{\bar{S}} \times \bar{i}_{no.eq.,sym} \quad (42a)$$

or

$$\bar{u}_{f,sym} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \frac{I''_{ph}}{Y''_{M1} + Y''_{M2} + Y''_{M0} + 3Y_f} \quad (42b)$$

which is the operationally dual expression to the case where an internal shunt short-circuit is observed

10 CONCLUSIONS

From this it must be stated that a network model, describing a synchronous machine connected to an arbitrary configured adjacent network can be set up using rather simple means in the description. Apart from making it possible to calculate the resulting short-circuit currents from shunt short-circuits (presuming that a proper modeling of the internal impedances can be made), it sets up the boundary conditions of how the internal impedances must be related.

An approach dual to that adopted for analyzing shunt short-circuits can be used for the treatment of internal series faults in synchronous machines. Thus, a method for calculating the induced voltage drops during series internal faults, using the Kronian organized mesh and cut-set methods has been demonstrated.

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