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USE A FUZZY INFERENCE TO ELECTRIC LOAD MODELING IN POWER DISTRIBUTION SYSTEMS

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Abstract. In electrical power distribution system, load modeling process is complicated because load system is usually monitored at only a few points. As a rule receiving nodes are not equipped with stationary measuring instruments so measurements of loads are performed sporadically. In general, the only information commonly available regarding loads, other than major distribution substations and equipment installations, is billing cycle customer kWh consumption. In order to model system uncertainty, inexactness, and random nature of customers' demand, a fuzzy system approach is proposed. This paper presents application possibilities of the fuzzy inference method to the electrical load modeling Clustering of load profiles in different part of system was used to classify the substations. A regression model, expressing the correlation between a substation peak load and a set of customer features (explanatory variables), existing in the substation population, is determined. Simulation studies have been performed to demonstrate the efficiency of the proposed scheme and an effect of different parameters on its accuracy on the basis of actual data obtained at distribution system substations.

Key Words: Load modeling, Fuzzy set theory, Fuzzy inference, Electrical power distribution systems.

1. Introduction

The knowledge of loads at system buses is one of the most important requirements for efficient operation of power distribution systems [1]. Estimation of loads is the basis for the system state estimation and for technical and economic calculations. This makes possible to improve operation and maintenance of electrical equipment and in planning of network operating configurations.

The main difficulties in the modelling of loads at receiving buses in distribution systems result from the random nature of loads, diversification of load shapes on different parts of the system, the deficiency of measured data and the fragmentary and uncertain character of information on loads and customers.

In the present stage of power distribution systems development, the mathematical estimation of the loads at the system buses seems to be the most realistic strategy due to incomplete primary information on loads and customers. It demands earlier determination of the stable relations between bus loads and easier available data [1].

The probabilistic models are widely used to estimate system loads. In order to develop the relevant types and parameters of probability distribution, large numbers of recorded consumption values are required. To obtain the above data a special measurement project has to be considered. The use of statistical methods is not always possible due to occurrence of a large deficit of measurements. The fuzzy set theory is a convenient mathematical tool that allows us partially to eliminate unreliability from input information and to limit the influence of deficit of measurements.

The most renowned method for expressing the uncertainty in load models is fuzzy set theory [2, 3, 4]. For the purpose of simplicity of mathematical operation the trapezoidal and triangular forms of fuzzy numbers are usually used (Fig. 1.).

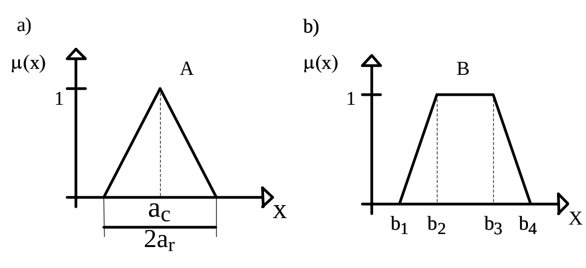

Fig. 1. Triangular and trapezoidal forms of fuzzy number

The paper presents application possibilities of the fuzzy set theory to electrical load estimation. Unreliable and inaccurate input data have been modelled by means of fuzzy numbers. A fuzzy inference model, expressing the correlation between a electrical load and a set of customer features (explanatory variables), existing in the substation population, is determined. Simulation studies have been performed to demonstrate the efficiency of the proposed scheme on the basis of actual data obtained at distribution system substations.

In practice the only information commonly available regarding loads in distribution substations is kWh consumption. In modeling process other input quantities also can be used (number of customers, rated power of transformers, installed capacity).

2. Fuzzy inference method

Many relationships between output quantities (peak load, load flow, losses of power and energy) and describing values coming from measurements can be represented by a fuzzy inference model (Fig. 2.).

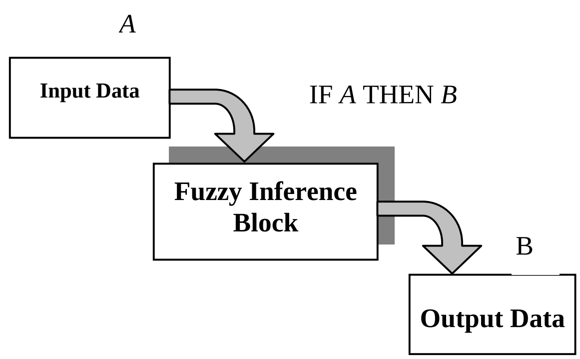

Fig. 2. Fuzzy inference model

A fuzzy expert system approach may be used to load curve prediction or peak load estimation in distribution system substations. In this case the fuzzy inference engine can process numerical and symbolic (linguistic) data. Input data can be form by using fuzzy c-means or hard clustering algorithms. The fuzzy inference block can be represented by multi-input single-output (MISO) model [5, 6]. In our problem the Takagi-Sugeno-Kang inference system was used. The TSK model is given by the following form (rules):

R_k: **IF** (z_1 is C_{k1}) **AND** … **AND** (z_n is C_{kn}) **THEN** f_k **is** $y_k(z_i)$

where:

k – number of rules;
z_i – input variables, $i=1, \dots, n$;
C_k – fuzzy sets;
f_k – output variables;
y_k – local linear function.

In this model local linear function can be represented by the linear fuzzy regression function, which can be described in the form presented in section 3.

3. Local fuzzy regression function

In this work fuzzy approach was used twice. First to identify input data in inference model, second to build adequate output models (local function). The general fuzzy regression model is given by the following equation [3]:

$$\tilde{\mathbf{Y}} = \mathbf{Z}\tilde{\mathbf{A}} \tag{1}$$

where:

$$\tilde{y}_i(z_i) = \tilde{a}_0 + \tilde{a}_1 z_{i1} + \cdots + \tilde{a}_k z_{ik} \qquad i = 1, 2, ..., n \tag{2}$$

The linear fuzzy regression model (1) is represented using symmetric triangular fuzzy parameters $\tilde{a}_i = [a_{ic}, a_{ir}]$ (Fig. 1) as follows:

$$\tilde{y}_i(z_i) = [a_{0c}, a_{0r}] + [a_{1c}, a_{1r}] z_{i1} + \cdots + [a_{kc}, a_{kr}] z_{ik} \tag{3}$$

$$y_{ci}(z_i) = a_{0c} + a_{1c} z_{i1} + \cdots + a_{kc} z_{ik} \tag{4}$$

$$y_{ri}(z_i) = a_{0r} + a_{1r} z_{i1} + \cdots + a_{kr} z_{ik} \tag{5}$$

where: y_c, a_c – center parameters of fuzzy numbers (membership function $\mu = 1$),
y_r, a_r – spreads of fuzzy numbers (geometrically the spread is a half of the base of the triangle).

The method uses the criterion of minimizing the total vagueness $J = f(y_r(z_i))$, defined as the sum of individual spreads of elements of vector $\tilde{\mathbf{Y}}$ [7].

$$J = y_{1r}(z_1) + y_{2r}(z_2) + \cdots + y_{nr}(z_n) \rightarrow \text{Minimum} \tag{6}$$

$$\text{subject to} \qquad y_i \in \tilde{\mathbf{Y}}(z_i), \qquad i = 1, 2, ..., n \tag{7}$$

$$a_{jr} \geq 0, \qquad j = 0, 1, 2, ..., k \tag{8}$$

Using equations (3) – (5) the problem can be written as follows:

$$J = \sum_{i=1}^{n} \left(a_{0r} + a_{1r}|z_{i1}| + \cdots + a_{kr}|z_{ik}| \right) \rightarrow \text{Minimum} \tag{9}$$

$$a_{0c} + \sum_{j=1}^{k} \left(a_{jc} z_{ij} \right) - (1-h) \left(a_{0r} + \sum_{j=1}^{k} \left(a_{jr} |z_{ij}| \right) \right) \leq y_i \qquad i = 1, 2, ..., n \tag{10}$$

$$a_{0c} + \sum_{j=1}^{k} \left(a_{jc} z_{ij} \right) + (1-h) \left(a_{0r} + \sum_{j=1}^{k} \left(a_{jr} |z_{ij}| \right) \right) \geq y_i \qquad i = 1, 2, ..., n \tag{11}$$

Dependence (9) presents a linear programming (LP) task, which can be easily solved by conventional techniques. The parameters $a_i = [a_{ic}, a_{ir}]$ of vector $\tilde{\mathbf{A}}$ are determined as the optimal solution of the LP problem (9) – (11).

4. Application of the clustering loads in substations to the electrical load modeling

The loads on distribution transformers are the instantaneous summations of the individual demands of many customers. Since the pattern of electrical demand of each customer cannot be determined precisely, it is usually necessary to calculate system loadings on an estimation basis.

Planning engineers use load modeling to predict the loads on different parts and different time of distribution systems. The load estimation can be done on the basis of the fuzzy inference with clustering method of customers load profiles [1].

The 24-hour load curves of the different substation groups show the characteristic variation for each group. Example daily load profiles for one substation are shown on Fig 3. There are three characteristic load values in the diagram:

- P_{dP} – the daily peak load,
- P_{dA} – the daily average load,
- P_{dB} – the daily base load.

On the basis of the analysis of the profiles from different days, the four characteristic intervals of the day are distinguished (Fig. 3.):

- night – **n**,
- morning – **m**,
- afternoon – **a**,
- evening – **e**.

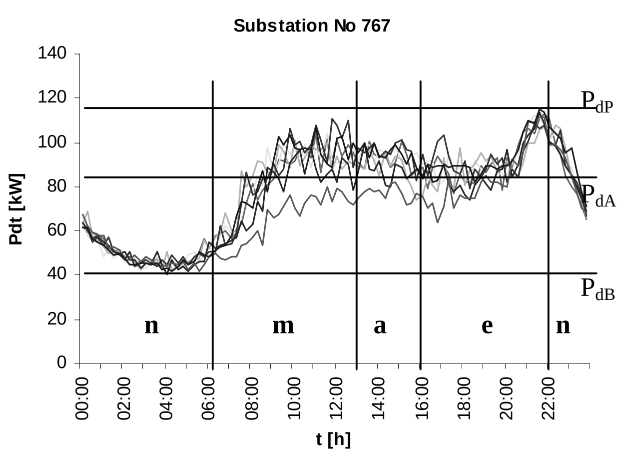

Fig. 3. Example of 24-hour load profiles for one week in substation No. 767

The clustering of the substations (customers) according to daily load curves is made on the base of the average alignment for each column. To avoid the influence of the instantaneous values on power changes consumed from substations, the average load of twenty four hours should be taken as reference quantity. The average alignment degree for each column is defined as ratio of the average load in the column to the daily average load:

$$l_c = \frac{P_{jA}}{P_{dA}} \tag{12}$$

where:

c – the column index (n, m, a, e),
P_{jA} – the average load in the column j,
P_{dA} – the daily average load.

The load diagrams are regarded as similar ones when their average alignment degrees for each column have similar values [1].

In practice the only commonly available information regarding loads, other than a major distribution substations, is kWh consumption. In modelling process other input quantities such as number of customers, rated power of transformers, and installed capacity can be used.

According to the above approach, a fuzzy regression model with the most important quantity (energy consumption – A_d) was prepared for different class of load.

The use of statistical methods is not always possible due to occurrence of a large deficit of measurements. The fuzzy set theory is a convenient mathematical tool that allows to partially eliminate unreliability from input information and to limit the influence of deficit of measurements. Additionally, use of inference method permits better adjustment of model to real behaviour of substations. The daily 15-minutes peak power consumption for a given substation may be found on the basis of input quantities using fuzzy inference process with local linear fuzzy regression models (2) and (9) - (11). The right selection of the investigating objects to build the fuzzy regression model is an important factor in load estimation process. The use of data from substations that belong to different classes can cause difficulties in working out a method to practical application. For this reason, it is necessary to compare all data characterizing substations especially attentively and to establish their membership to individual class of objects. In this work the clustering approach presented above was used in the fuzzy inference method to the electrical load modeling.

5. Numerical example

To verify the proposed method of peak load estimation the measurements of daily energy consumption A_d and daily peak load P_{dP} at selected distribution substations in Bialystok Power Distribution Utility Co. were made in July and August. Investigated objects are substations with transformers with 15/0.4 kV ratio of transformation and power ratio from 160 to 400 kVA. The frequency of measurement was 15 minutes. Description of substations and supplied customers are shown in table I.

TABLE I
DATA OF SUBSTATIONS AND SUPPLIED CUSTOMERS

Substation No.	Power of transformers	Number of customers	Type of customers
35	250 kV·A	50	commercial-services
158	250 kV·A	120	municipal-living
638	400 kV·A	150	municipal-living
734	250 kV·A	56	individual houses + small services
767	160 kV·A	148	municipal-living
1197	160 kV·A	67	music school

On the basis of above considerations in the first step clustering of load in substation was made. On the ground of measurements of the profiles average alignment degrees for each column was calculated. Example daily load profiles for substation No. 638 and No. 56 are shown on Fig. 4 and 5.

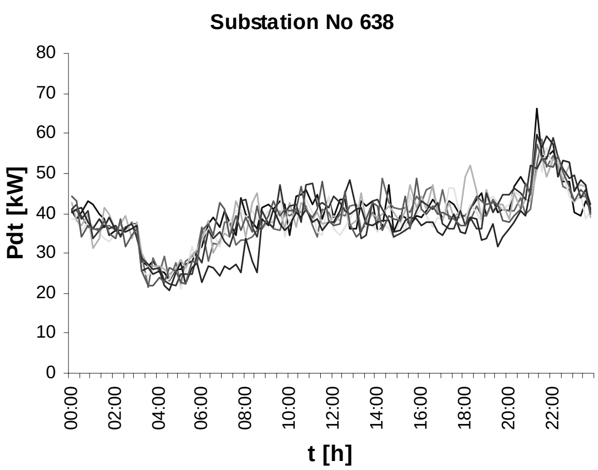

Fig. 4. Example of 24-hour load profiles for one week in substation No. 638

The clustering of load profiles on the plane l_e (the average alignment degree at the column **e** – evening) and l_m (the average alignment degree at the column **m** – morning) is shown on Fig. 6.

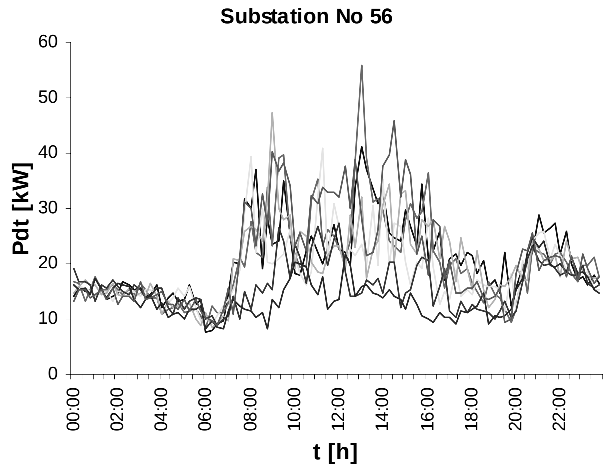

Fig. 5. Example of 24-hour load profiles for one week in substation No. 56

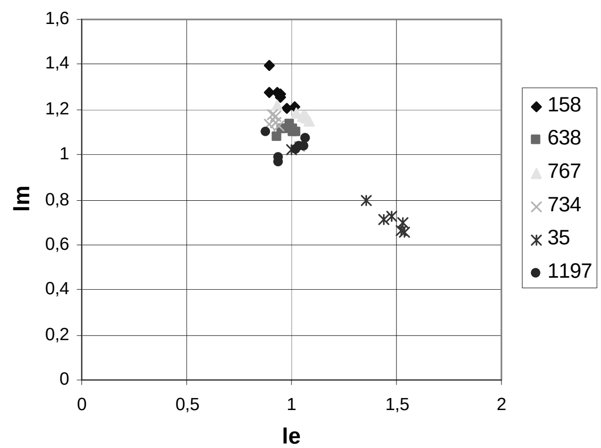

Fig 6. The example of clustering of daily profiles on the plane l_e and l_m for six substations

On the ground of measurements the general linear regression model (GLRM) and the general fuzzy linear regression model (GFLRM) with the most important quantity (A_d) were determined for substation No. 767. To build general model of daily peak load all type of days (working and weekend days) were used. The general fuzzy model can be presented in form:

$$P_{dP} = [a_{0c}, a_{0r}] + [a_{1c}, a_{1r}] \cdot A_d \qquad (13)$$

For this model the LP problem corresponding to the given data (Fig. 7) was formulated from (9) - (11). By solving the following model was obtained:

$$P^{I}_{dP} = [-25.4932, 0.0] + [0.0776, 0.0069] \cdot A_d \qquad (14)$$

In the next step for the same substation models for the selected two types of days (work and weekend days) were calculated. The LP problem corresponding to the given data (Fig. 8 and Fig. 9.) was formulated from (9) - (11). By solving this LP problem, the following fuzzy regression models are obtained:

- ♦ for substation No. 767 and work days

$$P^{I}_{ds} = [0.57687, 0.0] + [0.0606, 0.00398] \cdot A^{I}_{d} \qquad (15)$$

- ♦ for substation No. 767 and weekend days

$$P^{II}_{ds} = [14.5011, 0.0] + [0.0551, 0.0052] \cdot A^{II}_{d} \qquad (16)$$

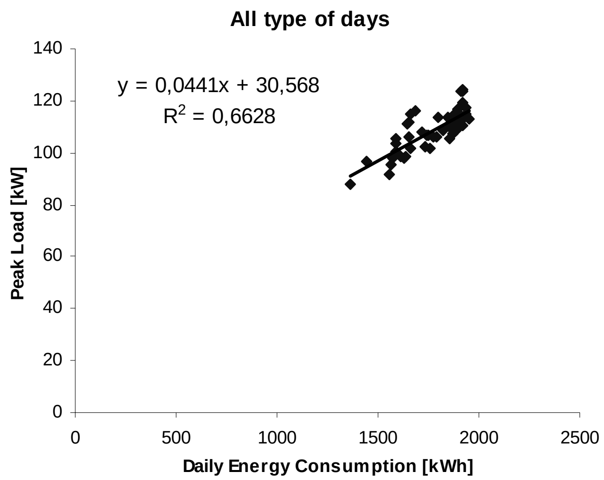

Fig. 7. The data set used for building general fuzzy regression model (14) for substation No. 767

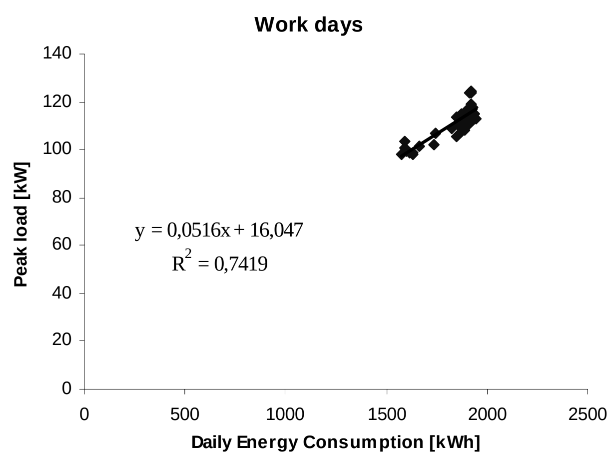

Fig. 8. The data set used for building fuzzy regression model for work days

On the basis of models (14) - (16) daily peak loads at the another substations in a period of three weeks in September were estimated. In the first step inference process was applied. On the basis of average alignment degree at morning and evening columns (Fig. 3) clustering of type of profiles was prepared. Taking advantage of consideration presented in sections II and III the two classes of load curves (Fig. 6) and two classes depending on type of day were constructed. In our experiment all substations belong to the same type of customers (excepting industrial customers).

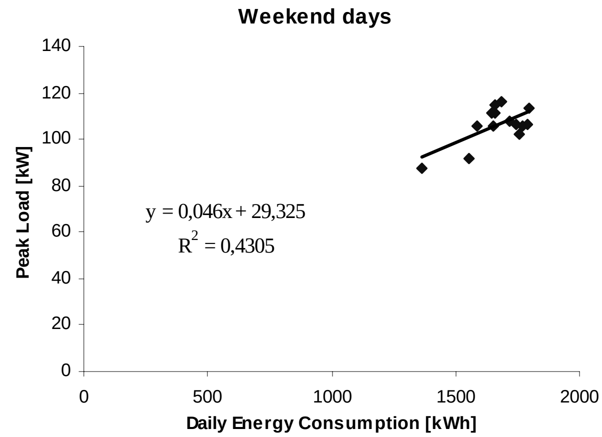

Fig. 9. The data set used for building fuzzy regression model for weekend days

The finally model looks as follows:

Rule 1: **IF** (profile *j* is similar as profile *767*) and (day is working) **THEN** use model I (form 15)

Rule 2: **IF** (profile *j* is similar as profile *767*) and (day is weekend) **THEN** use model II (form 16)

Rule 3: **IF** (profile *j* is not similar as profile *767*) and day is working or weekend **THEN** do not use any model

Considering the information presented above, four models were constructed for different classes of customers.

Load models with local fuzzy regression functions were verified on the basis of measurements in the substations No. 35, 158, 638, 734 and 1197 in September. Because substation No. 35 does not precisely belong to the class of load curve as substation 767 (Fig. 6), for calculations was used rule No 3 in inference model. The finally results of estimation process are shown in table II.

TABLE II
RESULT IN ESTIMATION PROCESS OF PEAK LOAD

Substation No. and Rule No.	Average absolute value of error in general regression model	Average absolute value of error in general fuzzy regression model	Average absolute value of error in local fuzzy regression model
767, Rule 1.	0,08921	0,03571	0,07650
638, Rule 1.	0,11679	0,24751	0,09949
1197, Rule 1.	0,36751	0,57696	0,15762
158, Rule 2.	0,12753	0,40493	0,07645
638, Rule 2.	0,1814	0,22912	0,08469
1197, Rule 2.	0,47739	0,6548	0,17164
35, Rule 3.	-	-	-

6. Conclusions

It result from the considerations and relationships described above that the fuzzy set approach to electrical load estimation adds a new quality into the system analysis in uncertain conditions.

The proposed method allows us to estimate daily peak power demand at distribution transformers during normal state conditions, on the basis of the energy consumption that is the most correlated factor with the peak load demand.

The presented results show that right selection of investigated objects have a large influence on the accuracy of proposed method. The errors of the fuzzy inference method depend on measurement experiment that allows to achieve better fit of model to real conditions.

The specification of the presented approach using day and season make the results more precise. Also the usage of clustering method gave interesting results.

The author sees usefulness of applying of fuzzy inference with clustering method and local fuzzy regression analysis to problem of load forecasting and load estimation in power distribution systems. The presented method may be a useful tool supporting planning distribution engineers.

The presented example illustrates the fuzzy inference approach in modelling of electrical peak loads.

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References

[1] J. Nazarko, *Modeling of Electrical Power Distribution System*s, Bialystok Technical University Publisher, Bialystok, 1993.

[2] M.E. El-Hawary, *Electric Power Application of Fuzzy Systems*. IEEE Press, Piscataway, NJ,1998.

[3] J. Nazarko, W. Zalewski, The Fuzzy Regression Approach to Peak Load Estimation in Power Distribution Systems. *IEEE Transactions on Power Systems*, Vol. 14, No. 3, August 1999, pp. 809-814.

[4] W. Zalewski, Combination of Fuzzy Regression Analysis and the Least Squares Regression Method to Modelling of Electrical Load, in *Proc. The 6th International Conference on Probabilistic Methods Applied to Power Systems PMAPS '2000*, Funchal, Portugal, September 25-28, 2000.

[5] R. R. Yager, D. P. Filev, *Essentials of Fuzzy Modelling and Control*. John Wiley & Sons, New York, 1994.

[6] P. Angelov, An Approach for Fuzzy Rule-base Adaptation Using On-line Clustering, *International Journal of Approximate Reasoning 35 (2004)*, pp. 275-289.

[7] H. Tanaka, H. Ischibuchi, Possibilistic Regression Analysis Based on Linear Programming in *Fuzzy Regression Analysis*, J. Kacprzyk, M. Fedrizzi Ed., Physica-Verlag, Heidelberg, 1992, pp. 47-60.

[8] S. Brandt, *Statistical and Computational Methods in Data Analysis*, Springer Verlag, New York, 1997.