## FAST AND OPTIMIZED LAGRANGIAN RELAXATION AND BUNDLING METHOD BASED THERMAL UNIT COMMITMENT AND ECONOMIC LOAD DISPATCH SCHEDULING IN LARGE SCALE POWER SYSTEMS

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#### ABSTRACT

This paper presents LaGrangian Relaxation (LR) and Bundling Method (BM) based Unit Commitment (UC) and Economic Load Dispatch (ELD) solution techniques. Through these techniques, thermal units consisting of 45 generators have been applied. A comparison has been made with a data taken from the giant power utility in Malaysia, Tenaga Nasional Berhad (TNB) and the results found from this system has been compared to that of TNB results. Substantial costs saving of 0.049% have been achieved. A very fast scheduling system for UC and ELD along the said costs saving has also been gained. These two achievements have been the two main objectives of this research.

#### **KEY WORDS**

Bundling Method, Economic Load Dispatch, Interior Point Method, LaGrangian Relaxation, LaGrangian Multipliers, LaGrangian Dual Problem

## 1. Introduction

Optimization problems in the power generation industry have attracted researchers, power producers and power experts for many years. The complex nature of generation of electricity implies ample opportunity of improvement towards the optimal power generation solution. The demand of power systems varies throughout the day and reaches a different peak values from one day to another. To satisfy this demand, to start-up and shutdown a number of generating units at various power stations each day is needed. Due to this, Unit Commitment (UC) and Economic Load Dispatch (ELD) problems play major role on finding an optimal power scheduling in electrical power system. The major problems are to decide when and which generating units to turn on and turn off and at the same time minimize the total fuel cost over specified period subject to a large number of difficult constraints.

The most important constraint is that the total generation must equal to the forecasted half-hourly, hourly, daily or weekly demands for electricity. Obtaining optimized scheduling for UC and ELD can considerably reduce the production costs, which is of increasing importance in the ongoing liberalization of the electricity markets in many countries [5].

The main problems involve modeling of UC and ELD and techniques used will be discussed in sections 2. Mathematical formulation of UC & ELD will be discussed in section 3. LR and BM as solving techniques with interior point method (IPM) for multipliers initialization will be discussed in section 4. Updating of LR multipliers will be presented in section 5. BM will be reviewed in section 6. Section 7 will highlight the results obtained from developed system using the selected techniques and compared with the TNB results. Section 8 will overview the concluding remarks of the overall performance of the system.

# 2. Unit Commitment and Economic Load Dispatch Problems

Unit Commitment (UC) aims at the selection of a generating unit to start-up and shut down in order to meet the forecasted demand. It is an operation scheduling function, and sometimes called pre-dispatch. To secure the operational requirements of a practical generation system, the commitment states (on/off) of hundreds of generating units should be provided for a time horizon from 24 hours to 168 hours. This commitment scheduling should maintain the balance between the generated power megawatt (MW) and the system demand under normal conditions. Moreover, sufficient MW spinning reserve should be available to account for the uncertainty of the demand and the generating unit failures.

Economic load dispatch (ELD) is basically the process of apportioning the total load on a system between the various generating plants to achieve the greatest economy of operation [6]. The actual power generation of each committed unit at every hour is calculated in the ELD part. The load demand required to meet varies in highly unpredictable fashion, and mostly nonlinear with the time. Due to this complexity, finding fast system and optimized cost load dispatch is very difficult to achieve.

Various techniques have been proposed and developed for solving the UC and ELD problems. These techniques are: Exhaustive Enumeration (EE) [2],[4], Priority List (PL) [1],[2],[4],[15],[16], Dynamic Programming (DP) [1],[2],[3],[4], Linear Programming (LP) [1],[2],[3],[4], Integer and Mixed Integer Programming (IP and MIP) [2],[3],[4], Branch and Bound (B&B) [2,4], Separable Programming. (SP) [2,4], Network Flow Programming (NF) [2],[4], Risk Analysis [2,4], Simulated Annealing (SAn) [2],[4], Augmented LaGrangian (AL) [4], Decision Analysis [2],[4], Genetic Algorithms (GA) [4],[15,], Relaxation LaGrangian (LR)[1],[2],[3],[4],[5],[7],[8],[9],[15] and so many others. As explained above, LR method is one of the most power techniques suitable for such this large-scale power system problems. For that reason, it has been applied in this research along with other techniques.

#### 3. UC and ELD Mathematical Formulation

The problem is to decompose the main problem into subproblems with specific periods during solution process. All known constraints and objective function are defined and formulated using the available and the selected techniques. The main objective of UC and ELD are to minimize the cost associated with power production in thermal system such as fuel costs, startup costs and shut down costs.

#### **3.1 UC Formulation**

For the objective function of unit commitment problem, LR can be expressed as:

Minimize z, z = 
$$\sum_{i=1}^{N} \sum_{j=1}^{T} \{ [F_i(P_i)] U_i^j + [Fsc_i] U_i^j \}$$
 (1)

Subject to :

$$\sum_{i=1}^{N} P_{i}^{j} \ge P_{D}^{j} + S_{R}^{j}$$
(2)

$$\sum_{i=1}^{N} \left( U_i^{j} P_i^{\max} \right) \ge P_D^{j} + S_R^{j}$$
(3)

$$U_i^{j} P_i^{\min} \le P_i^{j} \le U_i^{j} P_i^{\max}$$
(4)

$$\boldsymbol{x}_{i}^{j} \geq \boldsymbol{U}_{i}^{j} - \boldsymbol{U}_{i}^{j-1}; \quad \boldsymbol{x}_{i}^{j} \geq \boldsymbol{0}; \quad \boldsymbol{U}_{i}^{j} \geq \boldsymbol{0}$$
 (5) where

$$F_{i}(P_{i}) = a_{i} + b_{i}P_{i} + c_{i}P_{i}^{2}$$
(6)

is the fuel cost or running cost function and

$$Fsc_{i} = Cc_{i} \left( 1 - e^{\frac{-t}{\tau}} \right) \times Fl_{i} + Ch_{i} \left( Tsd_{i} \right) + Cf_{i}$$
(7)

is the start-up cost.

#### **3.2 ELD Formulation**

For ELD part, there are no on/off cases; instead the aim is to dispatch the load economically to meet the demand. In this case, the objective is to find the optimum generation cost of unit i and mathematically can be described as:

Optimize 
$$F = \sum_{i=1}^{N} F_i(P_i) + F_{sc_i}$$
 (8)

Subject to:

The system power balance:

$$\sum_{i=1}^{N} P_{i}^{j} \ge P_{D}^{j} + P_{Loss} + S_{R}^{j}$$
(9)

System reserve requirement:

$$\sum S_i^{\ j} \le k_i P_i^{\ j} \tag{10}$$

$$S_i^{j} \le P_i^{\max} - P_i^{j} \tag{11}$$

$$\sum S_i^j \le S_R^j \tag{12}$$

The generator capacity limit:

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{13}$$

Minimum uptime and downtime restrictions:

$$Tsd_i \ge Tsd_i^{\min} \tag{14}$$

$$Tup_{i} \ge Tup_{i}^{\min} \tag{15}$$

The loading and de-loading rate restrictions:

$$P_i^{J} - P_i^{J-1} \le \delta P_i^{+} T_j \tag{16}$$

$$P_i^{j} - P_i^{j-1} \le \delta P_i^{-T} T_j \tag{17}$$

The ramping rate limitations:

$$P_i^{j} - P_i^{j-1} \le Tru_i \tag{18}$$

$$P_i^j - P_i^{j-1} \le Trd_i \tag{19}$$

## 4. LaGrangian Relaxation Application

Through LR, the system power balance and the system reserve requirement are relaxed with Lagrange multipliers  $(\lambda^j \text{ and } \mu^j)$  respectively. Then, the LaGrangian dual function is formed by appending the relaxed coupling constraints with the objective function. The target here is to decompose the problem into original objective function and a new one using the balance and spinning reserve constraints.

#### 4.1 LR based UC

The relaxed minimum-maximum problem can be found by penalizing the load demand and spinning reserve. Adding new penalty terms to the objective function of the problem alters the constraints to the required limitations after iteration processes.

The problem can be expressed as:

$$\begin{aligned} \text{Maximize} & _{\lambda, \mu > 0} \boldsymbol{L}, \, \boldsymbol{L} = \sum_{j=1}^{r} \left( P_{D}^{j} \lambda^{j} + P_{D}^{j} \mu^{j} + S_{R}^{j} \mu^{j} \right) \\ + \min_{x \notin S} \sum_{j=1}^{T} \sum_{i=1}^{N} \left[ (b_{i} - \mu^{j} P_{i}^{\max}) U_{i}^{j} + (c_{i} - \lambda^{j}) P_{i}^{j} + Fsc_{i} \alpha_{i}^{j} \right] \end{aligned}$$

$$(20)$$

The inside minimization problem can be decomposed into single generator sub-problem. This can be formed as:

minimize 
$$_{x \notin S} \sum_{j=1}^{T} \sum_{i=1}^{N} \left[ (b_i - \mu^j P_i^{\max}) U_i^j + (c_i - \lambda^j) P_i^j + Fsc_i \alpha_i^j \right]$$

$$(21)$$

with the relationship of:

$$U_i^j P_i^{\min} \le P_i^j \le U_i^j P_i^{\max}$$
$$\alpha_i^j \ge U_i^j - U_i^{j-1}; \quad \alpha_i^j \ge 0; \quad U_i^j \ge 0$$

 $U_i^j$  is an integer for all i and j.

it can be simply formed by the following formulation: maximize  $F(\lambda, \mu)$  with all  $\mu^j \ge 0$ 

$$F(\lambda, \mu) = \text{minimize } \sum_{j=1}^{T} \left[ U_i^{\ j} F_i(P_i) + U_i^{\ j} (1 - U_i^{\ j-1}) Fsc_i(\alpha_i^{\ j}) - \lambda^j P_i^{\ j} - \mu^j U_i^{\ j} P_i^{\text{max}} \right]$$
(22)

#### 4.2 LR based ELD

The problem can be relaxed as: Maximize F ( $\lambda$ ,  $\mu$ ) with all  $\mu^{j} \ge 0$  where

$$F(\lambda^{j}, \mu^{j}) = \text{minimize}_{\lambda, \mu} \sum_{j=1}^{T} \left\{ \sum_{i=1}^{N} F_{i}(P_{i}) + Fsc_{i} - \lambda^{j} \left( \sum_{i=1}^{N} P_{i}^{j} - P_{D}^{j} + S_{R}^{j} + P_{LOSS} \right) - \mu^{j} \left( \sum_{i=1}^{N} P_{i}^{\text{max}} - \left( P_{D}^{j} + S_{R}^{j} \right) \right\}$$
(23)

subject to (9), (10), (11), (12), (13), (14), (15), (16), (17), (18) and (19).

#### 5. Updating the Multipliers

When applying LR, finding the fast updating multipliers are the main factor that can contribute to the fast solution to UC and ELD problems. Methods such as sub-gradient (SGM), bundling (BM) and cutting plane (CPM) are used to maximize the dual function. Those methods are based on the use of black box routine (simulator). A minimizing sequence is generated independently to compute the optimal value of the objective function.

SGM is the most commonly used technique among practitioners because of its simplicity of implementation. However, it progresses slowly to the optimum in an oscillating fashion [8],[9],[10],[11],[12],[13]. With a memory of past iteration, CPM defines a model of the objective function using past information [11]. CPM reconstructs the region of interest as well as other regions of no interest. The optimal value will be found by approaching the model to the objective function. The drawback occur on this method is how to define functions to defining the model.

Lately, BM is one of the best solution methods to LaGrangian dual problem [10],[11],12,[13],[14],[15],16]. With the information collected along iterations, BM constructs both models of the objective function and its sub-differential. This method improves the weakness of CPM. It avoids oscillations and keep-tracks the best point obtained so far along iteration to yield the final optimal value.

## 6. Bundling Method

Bundling method solve UC and ELD maximization problems using a concept called the  $\epsilon$ -subdifferential. It can be defined as :

$$\partial_{\varepsilon} f(x) = \left\{ g \in \mathbb{R}^n : f(y) \le f(x) + \left\langle g, y - x \right\rangle + \varepsilon, \forall y \in \mathbb{R}^n \right\}$$
(24)

 $\varepsilon$  is a set containing of all the information collected along iterations. Using this concept, BM accumulates all the function values in previous iteration into box of memory. It can be minimized by the following equation:

$$f_m = \min_{1 \le k \le n} \left[ L(\lambda) + g(\lambda^k)(\lambda - \lambda^k) \right]$$
(25)

Then, the minimum values from that box will be used to find the direction in order to update the current multipliers. The current multipliers will be compared with the updated multipliers to check for the convergence of the direction that has been selected. When the convergence of the direction is satisfied, the updated multipliers values will be used as current multipliers.

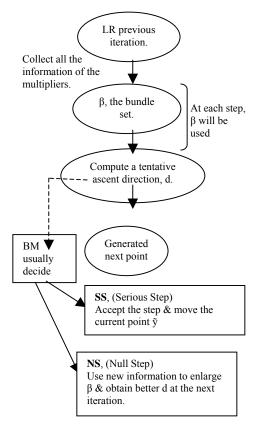


FIGURE 1: BASIC BUNDLE SOLUTION

### 7. Results and Discussions

To realize the costs optimization of UC and ELD and show fast scheduling of UC and ELD, the results achieved from the developed system has been presented in the following part. The developed system can contribute remarkable saving in the power generation systems. Verification of real world data has been done to prove the cost saving ability of the system. Several tests have been done using TNB real data. Table 1 presents the load demand at each hour in Peninsular Malaysia. Table 2 shows the input data of the TNB generation system. LR & BM methods have been implemented to the generation system, which consists of TNB's 45 units/generators. Table 3 presents the optimized power and costs results based on LR & BM solution in comparison with TNB output results.

Hour	Power	Hour	Power	Hour	Power
	Demand		Demand		Demand
	(MW)		(MW)		(MW)
1	5621	9	6972	17	7418
2	5583	10	7358	18	6810
3	5422	11	7605	19	6719
4	5286	12	7606	20	7034
5	5213	13	6858	21	7028
6	5518	14	7602	22	6640
7	5619	15	7562	23	6373
8	5872	16	7594	24	5849

TABLE 2: THE GENERATORS' INPUT DATA						
Unit	P <sub>min</sub>	P <sub>max</sub>	α (\$/h)	β	γ	
No.	(MW)	(MW)		(\$/MWh)	$(\$/MW^2h)$	
1	150	300	240.91	8.956	0.0012	
2	150	300	240.91	8.956	0.0012	
3	150	300	240.91	8.956	0.0012	
4	150	300	240.91	8.956	0.0012	
5	60	100	318.74	7.944	0.0109	
6	60	100	318.74	7.944	0.0109	
7	240	300	311.56	6.491	0.0007	
8	60	130	438.08	8.597	0.0001	
9	60	130	438.08	8.597	0.0001	
10	60	130	438.08	8.597	0.0001	
11	60	130	438.08	8.597	0.0001	
12	180	300	164.15	6.901	0.0021	
13	180	300	164.15	6.901	0.0021	
14	180	300	164.15	6.901	0.0021	
15	180	300	164.15	6.901	0.0021	
16	90	120	389.75	3.869	0.0356	
17	90	120	389.75	3.869	0.0356	
18	90	120	389.75	3.869	0.0356	
19	90	120	616.46	7.686	0.0575	
20	90	120	616.46	7.686	0.0575	
21	60	100	549.64	6.027	0.0331	
22	60	100	549.64	6.027	0.0331	
23	160	270	316.04	7.947	0.0107	
24	90	120	112.34	9.848	0.0012	
25	90	120	112.34	9.848	0.0012	
26	90	120	112.34	9.848	0.0012	
27	15	30	140.77	3.602	0.1349	
28	15	30	140.77	3.602	0.1349	
29	70	100	263.52	9.654	0.0016	
30	70	100	263.52	9.654	0.0016	
31	70	100	263.52	9.654	0.0016	
32	70	100	263.52	9.654	0.0016	
33	70	100	263.52	9.654	0.0016	
34	180	330	739.91	5.981	0.0002	
35	390	651	620.87	6.293	0.0013	
36	60	110	377.53	6.432	0.0207	
37	60	110	377.53	6.432	0.0207	
38	60	110	377.53	6.432	0.0207	
39	60	110	377.53	6.432	0.0207	
40	60	110	320.46	8.123	0.0098	
41	60	110	320.46	8.123	0.0098	
42	60	110	320.46	8.123	0.0098	
43	60	110	320.46	8.123	0.0098	
44	390	651	869.67	5.918	0.0006	
45	390	651	869.67	5.918	0.0006	
•	•				•	

TABLE 3: THE OPTIMIZED POWER OUTPUT AND COSTS USING LR IN COMPARISON WITH THE TNR OPTIMIZED POWER OUTPUT AND COST

COMPAR	COMPARISON WITH THE TNB OPTIMIZED POWER OUTPUT AND COSTS							
Unit No	Unit Status	Optimized Power, LR(MW)	Optimized Power, TNB(MW)	Optimize d Costs, LR (\$/hr)	Optimized Costs, TNB(\$/hr)			
1	1	150	270	1611.31	2746.51			
2	0	0	0	-	-			
3	1	150	250	1611.31	2554.91			
4	1	150	300	1611.31	3035.71			
5	0	0	0	-	-			
6	0	0	0	-	-			
7	1	300	300	2321.86	2321.86			
8	1	79.9996	60	1126.48	954.26			
9	1	79.9996	70	1126.48	1040.36			
10	1	79.9996	60	1126.48	954.26			
11	1	79.9996	104	1126.48	1333.25			
12	1	300	300	2423.45	2423.45			
13	1	300	300	2423.45	2423.45			
14	1	300	300	2423.45	2423.45			
15	1	300	300	2423.45	2423.45			

TABLE 2: THE GENERATORS' INPUT DATA

-		-			
16	1	90	110	1026.32	1246.10
17	0	0	0	-	-
18	1	90	110	1026.32	1246.10
19	1	90	110	1026.32	2157.67
20	1	90	110	1026.32	2157.67
21	0	0	0	-	-
22	0	0	0	-	-
23	1	160	260	1861.48	3105.58
24	0	0	0	-	-
25	0	0	0	-	-
26	0	0	0	-	-
27	0	0	0	-	-
28	0	0	0	-	-
29	0	0	0	-	-
30	0	0	0	-	-
31	1	70	100	947.14	1244.92
32	0	0	0	-	-
33	1	70	100	947.14	1244.92
34	1	330	330	2735.42	2735.42
35	1	651	645	5268.55	5220.69
36	0	0	0	-	-
37	0	0	0	-	-
38	0	0	0	-	-
39	0	0	0	-	-
40	0	0	0	-	-
41	0	0	0	-	-
42	0	0	0	-	-
43	0	0	0	-	-
44	1	651	362	4976.57	3090.61
45	1	651	362	4976.57	3090.61
Total		5213	5213	48668.9	51175.20

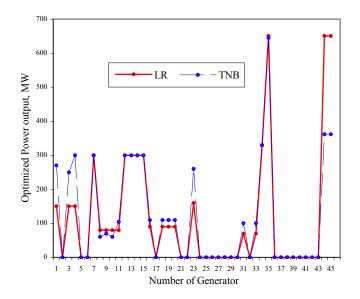
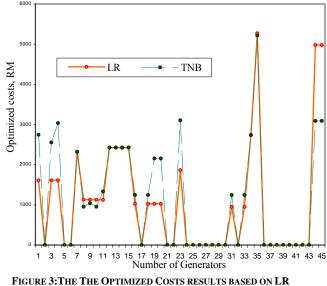


FIGURE 2: THE THE OPTIMIZED POWER RESULTS BASED ON LR SOLUTION IN COMPARISON WITH TNB OUTPUT RESULTS



IGURE 3: THE THE OPTIMIZED COSTS RESULTS BASED ON LK SOLUTION IN COMPARISON WITH TNB OUTPUT RESULTS

Figure 2 and 3 present the optimized power and costs results based on LR & BM solution in comparison with TNB output results. It also shows the capability of LR to solve large scale UC and ELD problems. It can be observed from the results that 0.049% saving in the total cost can be achieved in one hour. That means, for one whole day the total costs of more than 60,000 has been saved. The proposed techniques and approach give better results in terms of costs optimization and the results have been compared to the TNB results. In this case, it can be clearly proved that applying the optimal solution techniques can lead to remarkable cost optimization and increase profits for generation companies such as TNB.

Bundle method as a powerful approach for solving the lagrangian dual problems has been approved in previous work [10],[11],[12],[13],[14],[15],[16]. As shown below, Figure 4 and 5 present the short-term performance and long-term performance of a proposed new algorithm by Shih-Yih Lai and Ross Baldick [12]. In this work, the advantages of BM and SGM are combined together to update the multipliers. In Figure 4, the completion times of every iteration of the bundle algorithm and ALR+APP are shown as diamond and circles, respectively. It is shown that bundle algorithm and ALR+APP give fast iteration.

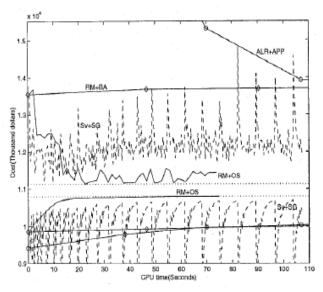


FIGURE 4: SHORT RUN PERFORMANCE

key for both figures:

RM+OS : Ramp Multipliers with OSS algorithm. Sv+SG : Svoboda et. al.'s approach with SGM RM+BA : Ramp multipliers with BM ALR+APP : Augmented LR with APP method

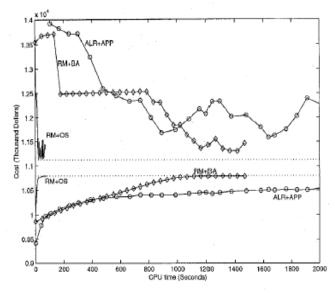


FIGURE 5: LONGER TERM PERFORMANCE

Figure 5 provides a comparison after bundle method reach its dual maximum. At that point, the ALR+APP still has not obtained any solution better than the bundle or the proposed algorithm. The other contribution of showing the advantages of the proposed Bundle Trust Region Method (BTRM) compared to SGM is done by Daoluan Zhang, Peter B. Luh and Yuanhui Zhang [11]. The BTRM and SGM are used to maximize the dual function with results summarized in table 4. It can be seen that when the units are different, SGM can converge to the optimal. BTRM converges with less number of function evaluations. It can also be seen that, when the units are identical, SGM has difficulty converging to the optimal, while BTRM converges easily. Through the results, it can be proved that BM has a good capability for solving the dual problems compared to the others.

	D.F.V. BTRM	N.F.E BTRM	D.F.V. SGM	N.F.E SGM	Optimal D.F.V.
Diff. U.	26600	6	26600	10	26600
Iden, U,	33000	5	32990	16	33000

Diff. U. – Different Units; Iden. U. – Identical Units; D.F.V.– Dual Function Value; N.F.E.– Number of Function Evaluations

## 8. Conclusion

This paper presents how LR and BM can successfully solve UC and ELD problems. Two important aspects for generation costs optimization and fast scheduling have been set as objective of this research and targeted to achieve them:, they are:

- 1) The reduction on the total operating costs of the generation units.
- 2) The reduction of computation time for the generation scheduling.

Results based on 45 generators have been presented using LR and BM methods. The results found also have been compared to the TNB results. Substantial cost saving of 0.049% have been achieved with fast computation time.

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