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TRANSFER FUNCTION ALGORITHM FOR LOCATING GROUND FAULTS ON POWER DISTRIBUTION

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ABSTRACT

A transfer function algorithm is proposed for locating line-to-ground faults on power distribution networks. This paper applies transfer function theory to the location of ground faults on power distribution networks. Transfer function equations are given for single-phase and three-phase distribution lines, with criteria for locating ground faults. Computer simulation shows that the criteria, which are based on the frequency, phase and shape characteristics of the transfer function spectrum, effectively locate faults in distribution networks. Since it is valid for power distribution networks with offset lines, and immune to load changes and its parameters can be easily measured, the fault location algorithm can be widely applied to pave distribution networks.

KEY WORDS: Power distribution networks, Fault location, Transfer function.

1. INTRODUCTION

In China, non-grounded type distribution networks are widely used in 10kV and 6kV transmission systems. However, single line-to-ground faults often occur in the distribution systems. Because of the radial structure of the tree-type offset lines, the limited measuring conditions available only in the transformer substation of the supply terminal, and the small effects on the electrical system caused by single line-to-ground faults, fault location in distribution networks is a rather difficult problem that remains unresolved. Various fault location and diagnosis techniques such as the Travelling Wave Fault Location Method and the Impedance Fault Location Method have been proposed in the literature. However, a survey of previous work has revealed that most of the fault location algorithms are developed for transmission systems and are not suitable for radial distribution networks. Other recent efforts like Port Ratio Branch Location Method and the On-line Detecting Method by Feeders are also not effective for distribution networks due to practicable problems. In the Chinese power administration bureau, the traditional distribution network fault location method is to select the fault line by switching off all offset lines in the transformer substation. After selecting the fault line, the operators have to find the ground fault point along the entire offset networks. To save time and resources and to maintain a continuous power supply, automatic methods are urgently needed to locate faults quickly and accurately.

Based on the analyses of various fault location methods, a novel fault location algorithm is proposed by using transfer function theory. This paper applies transfer function theory to ground fault location on power distribution networks, gives transfer function formulas for single-phase and three-phase distribution lines and describes the criterion for ground fault location. The frequency, phase and shape characteristics of the transfer function spectrum are shown to be effective criterion for fault location on distribution networks through numerical analysis and computer simulation of actual networks. Through a recursion calculation from the upper nodes of the tree type arrangement lines to the bottom nodes, the locating algorithm can be applied to distribution networks with offset lines. And because it selects the ground modal networks as the object of the analysis, the location algorithm is not affected by the changing of power load. Furthermore, the electrical system data needed for the algorithm can be obtained by making the measurement only in the supply terminal. All these advantages suggest that the location algorithm will be widely applied in the near future.

2. TRANSFER FUNCTION METHOD FOR FAULT LOCATION ON POWER DISTRIBUTION NETWORKS

The transfer function, defined as the ratio of the response in the frequency domain to the critical pulse, effects some basic properties of the system. This paper uses the transfer function for fault location in power networks.

Any distribution network system has a determinate structure. An impulse signal is imposed at one terminal of a selected main line, a fixed corresponding response can be detected at any terminal. When a ground fault occurs, the network structure will change and the response will differ for the same input signal. Furthermore, the responses will also differ for various fault locations in the network. The ratio of the input current to the input voltage is defined as the transfer function of a distribution network system. Criteria for fault location can be constructed from the characteristics of the transfer function to develop the transfer function method for fault location on power distribution networks. The method is accurate and is not affected by line voltage changes because the applied impulse is not in the same frequency range as the line frequency. The method is also very sensitive because the ground fault also affects the shape, phase and frequency of the transfer function which provides much information than a simple ratio. So the transfer function method for locating ground faults is innovative and practical.

2.1. Transfer function in single phase line

The distributed parameter lines model is introduced to analyze the response in the frequency domain. The series resistance, series inductance, shunt capacitance, and shunt conductance per unit length are denoted by *R,L,C* and *G* respectively. Functions *U(x,s) and I(x,s)*are the line voltage and current *x* km away from the starting terminal. The partial differential equations for the distribution system are:

$$-\frac{\partial u(x,t)}{\partial x} = Ri(x,t) + L\frac{\partial i(x,t)}{\partial t}, \tag{1}$$

$$-\frac{\partial i(x,t)}{\partial x} = Gu(x,t) + C\frac{\partial u(x,t)}{\partial t}, \tag{2}$$

A Laplace transformation,

$u(x,t)$ *to* $U(x,s)$ and $i(x,t)$ *to* $I(x,s)$,

Gives the voltage and current solutions in the frequency domain as:

$$U(x,s) = A_1(s)\mathrm{e}^{-\gamma(s)x} + A_2(s)\mathrm{e}^{\gamma(s)x}, \tag{3}$$

$$I(x,s) = \frac{1}{Z_c(s)}[A_1(s)\mathrm{e}^{-\gamma(s)x} - A_2(s)\mathrm{e}^{\gamma(s)x}], \tag{4}$$

where $Z_c(s) = \sqrt{\dfrac{R+sL}{G+sC}}$ is the characteristic impedance.

and $\gamma(s) = \sqrt{(R+sL)(G+sC)}$ is the propagation constant .

Assume that a ground fault occurs at the point l km away from the starting terminal with a fault resistance Z_0. If a rectangle impulse is then imposed at the base terminal, the boundary conditions in the frequency domain are

$$U(0,s) = \frac{1}{s} \text{ and } \frac{U(l,s)}{I(l,s)} = Z_0.$$

The solutions for these conditions are

$$\begin{aligned} U(x,s) = [&(Z_c(s)+Z_0(s))\mathrm{e}^{\gamma(s)(l-x)} \\ &-(Z_c(s)-Z_0(s))\mathrm{e}^{-\gamma(s)(l-x)}]/ \\ &\{s[(Z_c(s)+Z_0(s))\mathrm{e}^{\gamma(s)l} \\ &-(Z_c(s)-Z_0(s))\mathrm{e}^{-\gamma(s)l}]\} \end{aligned} \tag{5}$$

$$\begin{aligned} I(x,s) = [&(Z_c(s)+Z_0(s))\mathrm{e}^{\gamma(s)(l-x)} \\ &+(Z_c(s)-Z_0(s))\mathrm{e}^{-\gamma(s)(l-x)}]/ \\ &\{Z_c(s)s[(Z_c(s)+Z_0(s))\mathrm{e}^{\gamma(s)l} \\ &-(Z_c(s)-Z_0(s))\mathrm{e}^{-\gamma(s)l}]\} \end{aligned} \tag{6}$$

The transfer function is then formulated as

$$\begin{aligned} T(s) &= \frac{I(x,s)}{U(x,s)}\Big|_{x=0} \\ &= \frac{Z_0(s)\mathrm{sh}\gamma(s)l + Z_c(s)\mathrm{ch}\gamma(s)l}{Z_c(s)[Z_0(s)\mathrm{ch}\gamma(s)l + Z_c(s)\mathrm{sh}\gamma(s)l]} \\ &= \frac{1}{Z_c(s)}\mathrm{th}[\sigma(s)+\gamma(s)l], \end{aligned} \tag{7}$$

where $\mathrm{th}\sigma(s) = \dfrac{Z_c(s)}{Z_0(s)}$.

2.2. Transfer function in three-phase lines

In three-phase lines, the electromagnetic complex coupling between lines requires the use of phase-modal transformation theory.

2. 2.1. Transfer function for balanced lines

The lines are assumed to be balanced or perfectly transposed ones. In this case the partial differential equations for the system are:

$$-\frac{\partial \boldsymbol{U}(x,t)}{\partial x} = \boldsymbol{RI}(x,t) + \boldsymbol{L}\frac{\partial \boldsymbol{I}(x,t)}{\partial t}, \tag{8}$$

$$-\frac{\partial \boldsymbol{I}(x,t)}{\partial x} = \boldsymbol{GU}(x,t) + \boldsymbol{C}\frac{\partial \boldsymbol{U}(x,t)}{\partial t}, \tag{9}$$

$$\boldsymbol{U}(x,t) = \begin{bmatrix} u_a(x,t) \\ u_b(x,t) \\ u_c(x,t) \end{bmatrix}, \quad \boldsymbol{I}(x,t) = \begin{bmatrix} i_a(x,t) \\ i_b(x,t) \\ i_c(x,t) \end{bmatrix}$$

$$\boldsymbol{R}=\begin{bmatrix} r_0 & 0 & 0 \\ 0 & r_0 & 0 \\ 0 & 0 & r_0 \end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix} g_0 & 0 & 0 \\ 0 & g_0 & 0 \\ 0 & 0 & g_0 \end{bmatrix}$$

$$\boldsymbol{L}=\begin{bmatrix} l_d & l_{od} & l_{od} \\ l_{od} & l_d & l_{od} \\ l_{od} & l_{od} & l_d \end{bmatrix},\quad \boldsymbol{C}=\begin{bmatrix} c_d & c_{od} & c_{od} \\ c_{od} & c_d & c_{od} \\ c_{od} & c_{od} & c_d \end{bmatrix}$$

A Laplace transformation,

$\boldsymbol{U}(x,t)$ *to* $\boldsymbol{U}(x,s)$ and $\boldsymbol{I}(x,t)$ *to* $\boldsymbol{I}(x,s)$,

can be used to write the equations as:

$$\frac{\mathrm{d}^2\boldsymbol{U}(x,s)}{\mathrm{d}x^2}=\boldsymbol{PU}(x,s), \tag{10}$$

$$\frac{\mathrm{d}^2\boldsymbol{I}(x,s)}{\mathrm{d}x^2}=\boldsymbol{PI}(x,s), \tag{11}$$

where $\boldsymbol{P}$ is a balanced matrix with the elements formulated as follows:

$$p_{ii}=p_d=(l_d\ s+r_0)(c_d\ s+g_0)+2l_{od}c_{od}\ s^2$$
$$(i=1,2,3)$$
$$p_{ij}=p_{od}=(l_d\ s+r_0)c_{od}\ s+(c_d\ s+g_0)l_{od}\ s$$
$$+l_{od}c_{od}\ s^2(i,j=1,2,3,i\neq j)$$

A modal transformation,

$\boldsymbol{U}(x,s)=\boldsymbol{SU}_m(x,s)$ and $\boldsymbol{I}(x,s)=\boldsymbol{SI}_m(x,s)$,

can be used to write equation (12) and (13) as follows:

$$\frac{\mathrm{d}^2\boldsymbol{U}_m(x,s)}{\mathrm{d}x^2}=\boldsymbol{S}^{-1}\boldsymbol{PSU}_m(x,s)=\boldsymbol{\Lambda U}_m(x,s), \tag{12}$$

$$\frac{\mathrm{d}^2\boldsymbol{I}_m(x,s)}{\mathrm{d}x^2}=\boldsymbol{S}^{-1}\boldsymbol{PSI}_m(x,s)=\boldsymbol{\Lambda I}_m(x,s), \tag{13}$$

where $\boldsymbol{U}_m(x,s)=(U_0(x,s),U_\alpha(x,s),U_\beta(x,s))^T$

$$\boldsymbol{I}_m(x,s)=(I_0(x,s),I_\alpha(x,s),I_\beta(x,s))^T$$

$$\boldsymbol{\Lambda}=\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix},\ \boldsymbol{S}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\boldsymbol{S}^{-1}=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\lambda_1=p_d+2p_{od},\ \lambda_2=\lambda_3=p_d-p_{od}$$

So the modal voltage equations are

$$\frac{\mathrm{d}^2U_0(x,s)}{\mathrm{d}x^2}=\gamma_0{}^2(s)U_0(x,s), \tag{14}$$

$$\frac{\mathrm{d}^2U_\alpha(x,s)}{\mathrm{d}x^2}=\gamma_\alpha{}^2(s)U_\alpha(x,s), \tag{15}$$

$$\frac{\mathrm{d}^2U_\beta(x,s)}{\mathrm{d}x^2}=\gamma_\alpha{}^2(s)U_\beta(x,s), \tag{16}$$

where

$$\gamma_0{}^2(s)=(l_d+2l_{od})(c_d+2c_{od})s^2$$
$$+[r_0(c_d+2c_{od})+g_0(l_d+2l_{od})]\ s+r_0g_0$$
$$\gamma_\alpha{}^2(s)=(l_d-l_{od})(c_d-c_{od})s^2$$
$$+[r_0(c_d-c_{od})+g_0(l_d-l_{od})]\ s+r_0g_0$$

The solutions of the modal voltage equations are

$$U_0(x,s)=A_1(s)\mathrm{e}^{-\gamma_0(s)x}+A_2(s)\mathrm{e}^{\gamma_0(s)x}, \tag{17}$$

and

$$U_\alpha(x,s)=U_\beta(x,s)=B_1(s)\mathrm{e}^{-\gamma_\alpha(s)x}$$
$$+B_2(s)\mathrm{e}^{\gamma_\alpha(s)x}, \tag{18}$$

In the modal domain,

$$\boldsymbol{I}_m(x,s)=-\boldsymbol{S}^{-1}\boldsymbol{B}^{-1}\boldsymbol{S}\frac{\mathrm{d}\boldsymbol{U}_m(x,s)}{\mathrm{d}x}, \tag{19}$$

where $\boldsymbol{B}$=$\boldsymbol{R}$+s$\boldsymbol{L}$,

Let $\boldsymbol{F}=\boldsymbol{B}^{-1}=\begin{bmatrix} f_d & f_{od} & f_{od} \\ f_{od} & f_d & f_{od} \\ f_{od} & f_{od} & f_d \end{bmatrix}$

so

$$f_d=[(l_d-l_{od})s^2+2l_dr_0\ s+r_0{}^2]\ /$$
$$[(l_d{}^3-3l_dl_{od}{}^2+2l_{od}{}^3)s^3$$
$$+3r_0(l_d{}^2-l_{od}{}^2)s^2+3l_dr_0{}^2\ s+r_0{}^3]$$
$$f_{od}=[-l_{od}(l_d-l_{od})s^2-l_{od}r_0\ s]\ /$$
$$[(l_d{}^3-3l_dl_{od}{}^2+2l_{od}{}^3)s^3$$
$$+3r_0(l_d{}^2-l_{od}{}^2)s^2+3l_dr_0{}^2\ s+r_0{}^3]$$

The modal current solutions are:

$$I_0(x,s)=-(f_d+2f_{od})\frac{\mathrm{d}U_0(x,s)}{\mathrm{d}x}, \tag{20}$$

$$I_\alpha(x,s)=I_\beta(x,s)=-(f_d-f_{od})\frac{\mathrm{d}U_\alpha(x,s)}{\mathrm{d}x}, \tag{21}$$

Assume that the ground fault occurs at a location l km away from the base terminal in phase A with a grounded resistance Z_0. If a rectangular impulse is imposed on the lines at the base terminal, the boundary conditions applied to the lines are：

$$U_a(0,s)=U_b(0,s)=U_c(0,s)=\frac{1}{s},$$

$$I_a(l,s)=\frac{U_a(l,s)}{Z_0(s)},\ I_b(l,s)=I_c(l,s)=0.$$

The corresponding boundary conditions in the modal domain are:

$$U_m(0,s)=S^{-1}U(0,s)=(\frac{1}{s},0,0)^T$$

$$U(l,s)=SU_m(l,s)$$
$$=\begin{bmatrix}U_0(l,s)+U_\alpha(l,s)+U_\beta(l,s)\\U_0(l,s)-2U_\alpha(l,s)+U_\beta(l,s)\\U_0(l,s)+U_\alpha(l,s)-2U_\beta(l,s)\end{bmatrix}$$

$$I_m(l,s)=S^{-1}I(l,s)=\frac{1}{3Z_0(s)}\begin{bmatrix}U_a(l,s)\\U_a(l,s)\\U_a(l,s)\end{bmatrix}$$
$$=\frac{1}{3Z_0(s)}\begin{bmatrix}U_0(l,s)+U_\alpha(l,s)+U_\beta(l,s)\\U_0(l,s)+U_\alpha(l,s)+U_\beta(l,s)\\U_0(l,s)+U_\alpha(l,s)+U_\beta(l,s)\end{bmatrix}$$

Substitute these conditions into equations (17), (18), (19) and (20), the ground modal voltage and current can be obtained as:

$$U_0(x,s)=A_1(s)e^{-\gamma_0(s)x}+[\frac{1}{s}-A_1(s)]e^{\gamma_0(s)x},\quad(22)$$

$$I_0(x,s)=(f_d+2f_{od})\gamma_0(s)\{A_1(s)e^{-\gamma_0(s)x}$$
$$-[\frac{1}{s}-A_1(s)]e^{\gamma_0(s)x}\},\quad(23)$$

The transfer function equation in three phases is

$$T(s)=\frac{I_0(0,s)}{U_0(0,s)}$$
$$=(2A_1(s)s-1)\gamma_0(s)(f_d+2f_{od}),\quad(24)$$

where

$$A_1(s)=\frac{D_1(s)}{D_2(s)},$$
$$D_1(s)=e^{\gamma_0(s)l}\{[3Z_0(s)\gamma_0(s)(f_d+2f_{od})$$
$$+1]\gamma_\alpha(s)(f_d-f_{od})\mathrm{ch}\gamma_\alpha(s)l$$
$$+2\gamma_0(s)(f_d+2f_{od})\mathrm{sh}\gamma_\alpha(s)l\},$$
$$D_2(s)=2s\{[3Z_0(s)\gamma_0(s)(f_d+2f_{od})\mathrm{ch}\gamma_0(s)l$$
$$+\mathrm{sh}\gamma_0(s)l]\gamma_\alpha(s)(f_d-f_{od})\mathrm{ch}\gamma_\alpha(s)l$$
$$+2\gamma_0(s)(f_d+2f_{od})\mathrm{ch}\gamma_0(s)l\cdot\mathrm{sh}\gamma_\alpha(s)l\}.$$

2.2.2. Transfer function for untransposed lines

The partial differential equations for the system have the same form to untransposed lines. But the parameters' matrixes are changed to :

$$U(x,t)=\begin{bmatrix}u_a(x,t)\\u_b(x,t)\\u_c(x,t)\end{bmatrix},\quad I(x,t)=\begin{bmatrix}i_a(x,t)\\i_b(x,t)\\i_c(x,t)\end{bmatrix}$$

$$R=\begin{bmatrix}r_0&0&0\\0&r_0&0\\0&0&r_0\end{bmatrix},\quad G=\begin{bmatrix}g_0&0&0\\0&g_0&0\\0&0&g_0\end{bmatrix}$$

$$L=\begin{bmatrix}l_a&l_{ab}&l_{ac}\\l_{ab}&l_b&l_{bc}\\l_{ac}&l_{bc}&l_c\end{bmatrix},\quad C=\begin{bmatrix}c_a&c_{ab}&c_{ac}\\c_{ab}&c_b&c_{bc}\\c_{ac}&c_{bc}&c_c\end{bmatrix}$$

A Laplace transformation,

$$U(x,t)\ \ to\ \ U(x,s)\ \text{and}\ I(x,t)\ \ to\ \ I(x,s),$$

can be used to write the equations as follows:

$$\frac{d^2U(x,s)}{dx^2}=PU(x,s),\quad(25)$$

$$\frac{d^2I(x,s)}{dx^2}=P^TI(x,s),\quad(26)$$

where

$$P=(R+sL)(G+sC),P^T=(G+sC)(R+sL)$$

For L and C are not balanced, $P\neq P^T$.
A modal transformation,

$$U(x,s)=SU_m(x,s)\ \text{and}\ I(x,s)=SI_m(x,s),$$

can be used to write equation (27) and (28) as :

$$\frac{d^2U_m(x,s)}{dx^2}=Q^{-1}PQU_m(x,s)=\Lambda U_m(x,s),\quad(27)$$

$$\frac{d^2I_m(x,s)}{dx^2}=S^{-1}P^TSI_m(x,s)=\Lambda I_m(x,s),\quad(28)$$

Where S and Q are dependent on the configuration of the untransposed lines, and S=Q^T. Then

$$S^{-1}P^TS=Q^TP^TQ^{-T}=(Q^{-1}PQ)^T=\Lambda$$

and

$$U_m(x,s)=(U_0(x,s),U_\alpha(x,s),U_\beta(x,s))^T$$
$$I_m(x,s)=(I_0(x,s),I_\alpha(x,s),I_\beta(x,s))^T$$

$$\Lambda=\begin{bmatrix}\lambda_0^2(s)&&\\&\lambda_\alpha^2(s)&\\&&\lambda_\beta^2(s)\end{bmatrix}$$

So the modal voltage equations are :

$$\frac{\mathrm{d}^2 U_0(x,s)}{\mathrm{d}x^2} = \lambda_0{}^2(s) U_0(x,s), \tag{29}$$

$$\frac{\mathrm{d}^2 U_\alpha(x,s)}{\mathrm{d}x^2} = \lambda_\alpha{}^2(s) U_\alpha(x,s), \tag{30}$$

$$\frac{\mathrm{d}^2 U_\beta(x,s)}{\mathrm{d}x^2} = \lambda_\beta{}^2(s) U_\beta(x,s), \tag{31}$$

The remaining process is similar to that of the balanced lines, we can get the transfer function finally. And the matrixes ***S*** and ***Q*** are uncertain, and so, by the similar arguments, we can get the transfer functions for untransposed lines.

2.3. FAULT LOCATION CRITERION

2.3.1. The criterion

To locate the fault, a criterion related to fault distance must be abstracted from the characteristics of the transfer function. For a single phase line, equation (7) can be simplified by assuming a no-loss line.

$$Z_c(s) = \sqrt{\frac{R+sL}{G+sC}} \approx \sqrt{\frac{L}{C}}$$

$$\gamma(s) = \sqrt{(R+sL)(G+sC)} \approx s\sqrt{LC}$$

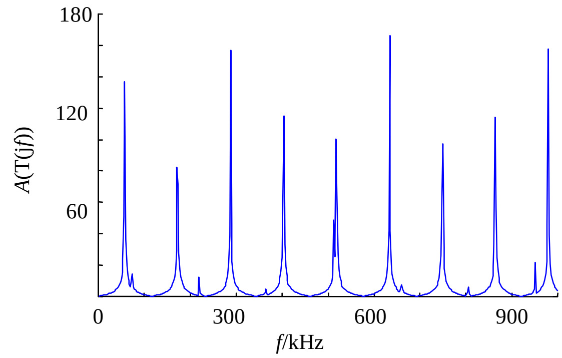
For high frequencies, $\sigma(s)$ is neglected, equation (7) becomes

$$T(j\omega) = -\frac{\mathrm{j}}{Z_c}\mathrm{tg}\,\omega\sqrt{LC}\,l$$

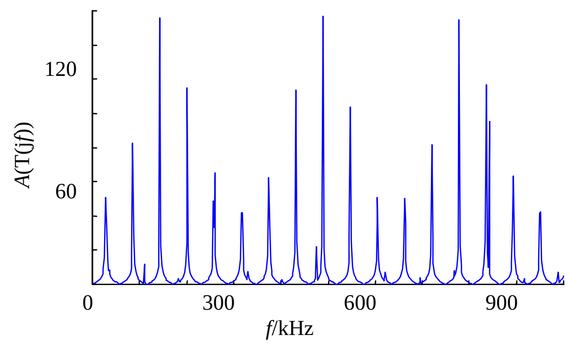
An analysis of the simplified formula indicates that the transfer function spectrum is periodic on the frequency axis with a period of $f_T = \frac{c}{2l}$,

where $c = 3\times10^5$ km/s is the propagation rate of electromagnetic waves; f_T is the period of the transfer function on the frequency axis in Hz; l is the distance of the ground fault from the base terminal in km. The fault location criterion can therefore be determined from the frequency characteristics of the transfer function spectrum.

The transfer function in three phase line, Eq.(23) can not be easily simplified. The system is analyzed numerically. The parameter matrixes of R, L, C and G are determined for actual distribution lines selected for the research. The time step was set to 1 μS with 500 sampling point. The Laplace operator is $s=j\times 2\pi\times 10^6$ Hz. Equation (23) was then solved to produce the transfer function amplitude spectrum shown in figure 1, where part (a) and (b) represent fault points located 1 km and 2 km from the base point, respectively.



(a) 1 km



(b) 2 km

Fig. 1. Numerically determined transfer function spectrum for different fault distances

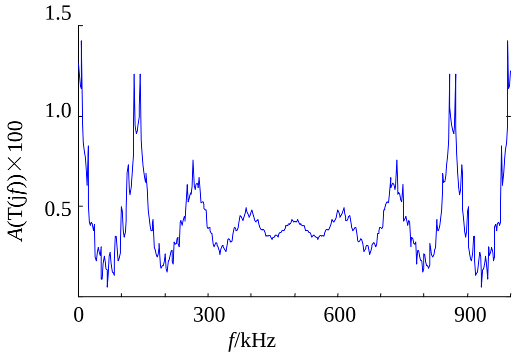
The numerical analysis of transfer function in three phase lines has the following characteristics:

(1) The transfer function spectrum exhibits uniformly spaced waves;
(2) The distance between wave crests along the frequency axis is inverse proportional to the fault distance, so the fault distance can be calculated from the wave crest separator as the fault location criterion.

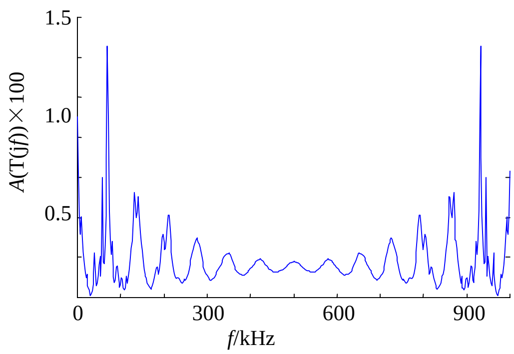
2.3.2. Verification of the criterion by simulation

EMTP simulation of a distribution line from a transformer substation with no offsets is selected to verify the criterion. The distributed line parameters are input to the EMTP program. The frequency spectrum analyses of the calculated results give the amplitude spectrums of the transfer function for lines with fault distances of 1 km and 2 km, Figure 2.

The EMTP simulation results show transfer function spectrums for fault lines exhibit uniformly spaced periodical waves, indicating that the transfer function algorithm for ground fault location on power distribution networks is effective.



(a) Fault distance of 1 km



(b) fault distance of 2 km

Fig. 2. Simulated transfer function spectrum of lines with different fault distances

The EMTP simulation results show transfer function spectrums for fault lines exhibit uniformly spaced periodical waves, indicating that the transfer function algorithm for ground fault location on power distribution networks is effective.

2.3.3. Discussion at the criterion

(1) Interpretation of the difference between the calculated results and the simulation. From the contrast between the Figure 1(a) and Figure 2(a), some difference can also be found besides the similarity in the main principle. Because the figure of simulation calculation is obtained by digital signal processing method of FFT, that the windows function multiply the sampling data leads the differences in characteristic of figure. The differences are: (i) at the crest of each wave, the simulation result is smooth while the transfer function is sharp and (ii) some small peaks appear in the simulation due to the boundary windows derived from FFT analysis. All these influences from the FFT analysis can be reduced by selecting an appropriate windows function and filter algorithm.

(2) Criteria can be developed to identify faults in distribution lines with offsets by combining the frequency and phase characteristics of the figure. A recursive analysis from the main node to the other nodes can give the fault offset and the fault distance. A detailed example will be given to illustrate the location processing.

A typical distribution network with offsets, Figure 3, has *M* as the base node whose measurements occur, *A* and *B* are nodes with offsets, and *C*, *D* and N are nodes at the end of lines with equal loads. The length of every offset is showed in Figure 3. A ground fault is assumed to occur on the *AB* offset with the fault point 2 km from *A*. The rectangular pulse is applied at the node *M*, which is the coordinate origin. The measured voltage and current at node *M*, $u(0,t)$ and $i(0,t)$, can be used to calculate the voltage and current of any node in the line, $u(x,t)$ and $i(x,t)$, by using the Bergerion algorithm and the distributed parameter line equation. The transfer function in any offset of the line can be calculated and analyzed.

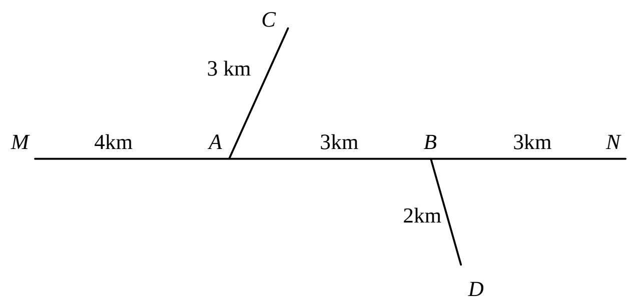
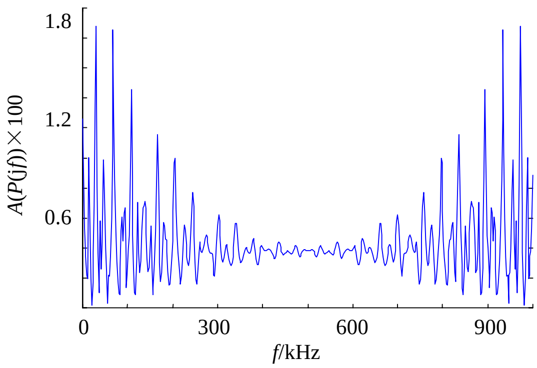


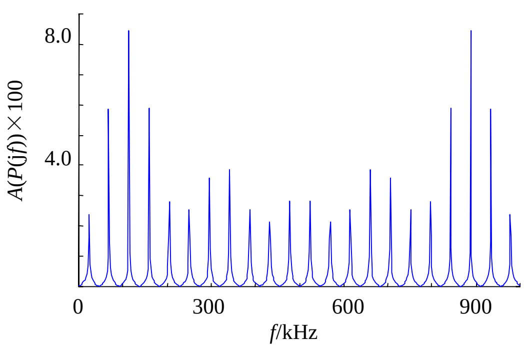
Fig. 3. An example of distribution networks with offsets

The transfer function for node *M*, $P_{MA}(s)$, is calculated first to determine whether the fault is located in the offset *MA*. Experience has shown that if the fault is located in the offset *MA*, the transfer function for the offset *MA* has uniformly spaced waves, shows in Figure 4(a). If the transfer function does not have the principle, the offsets are analyzed recursively step by step.

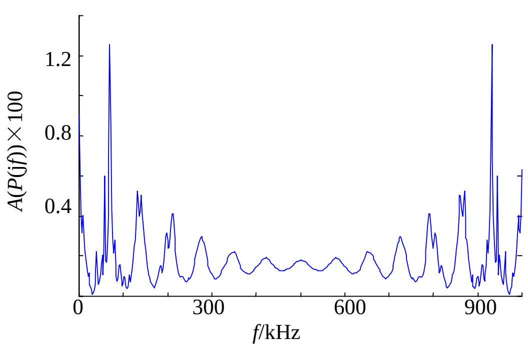
From the calculated voltage and current at node *A*, $P_{AB}(s)$ and $P_{AC}(s)$, which are the transfer function of offsets *AB* and *AC*, are given in Figures 4(b) and 4(c). Analysis of various results has shown that for the offsets at one node, the offsets without the grounded fault are equivalent to an open circuit and the offset containing ground fault is equivalent to a line with a grounded resistance at the end of the line. A phase difference of $\pi/2$ occurs between the two types of lines. By comparing the phases of all the offsets at one node, the offset with the fault can be identified and the analysis can proceed recursively to the offset with the fault. For this example, comparing the phases of transfer functions in Figure 4(b) and 4(c) show that offset *AC* is an open circuit line, so the recursive calculation should analyze offset *AB*. Furthermore, since the peaks are uniformly distributed in Fig. 4c, offset *AB* contains the ground fault. The fault distance, calculated from a distance measuring formula of this example, is 1.88 km away from node *A*.



(a) Spectrum of offset *MA*



(b) Spectrum of offset A*C*



(c) Spectrum of offset *AB*

Fig. 4. Transfer function spectrum for offsets *MA*, *AB* and *AC* for fault located in offset AB

Location results for faults occurring in other locations are given in Table 1.

More complex distribution networks with more offsets can be analyzed in the same way. Therefore, the criterion derived from the combined information of the shape, frequency and phase characteristics of the transfer function in all the offsets can be effectively used to locate ground faults in practical distribution networks.

Tab. 1. Sample of fault location in a distribution network with offsets, Fig. 3

Example	Ground resistance R / Ω	Assumed fault offset
1	10.0	*MA*
2	10.0	*AC*
3	10.0	*AB*
4	10.0	*BN*

Assumed fault distance l /km	Calculated fault offset	Calculated fault distance l /km
2.0	*MA*	1.8844
1.0	*AC*	1.0760
2.2	*AB*	2.1134
1.3	*BN*	1.3841

(3) Formula for determining fault distance. For distribution lines with the uniform parameters, formula can be easily obtained for determining fault distance. For any offset in a network, the fault distance is approximately proportional to the reciprocal of the distance between peaks. Therefore, the formula for determining the fault distance is

$$l(\text{km}) = K(\text{km/s})\frac{1}{f_{\mathrm{T}}(\text{Hz})} ,$$

where l is the fault distance and f is the distance between peaks. The coefficient K can be calculated from several sample calculations with assumed faults at several different locations. K is different for lines with different parameters and must be recalculated for each line. In the example of this paper, $K = 1.346\times10^{5}$ km/s . For non-linear situations in practical networks, the formula can be replaced by a fault dictionary to provide more exact locations.

(4) Insensitivity to load. Since transfer function algorithm for locating ground faults uses the ground modal networks, the algorithm is not affected by load changes. The algorithm can be used by just measuring data in one line terminal at the transformer substation, so it does not need to communicate and detect continuously. Therefore, the method is of great practicable use. The main work in the fault locating algorithm lies in the digital signal processing needed to calculate the transfer function peaks.

3. CONCLUSION

(1) A transfer function algorithm is developed for locating ground faults on power distribution networks. The transfer function theory is applied to locate ground faults on single phase and three phase distribution networks. A criterion is presented for locating ground

faults. Simulation shows that the criterion based on the frequency, phase and shape characteristics of the transfer function frequency spectrum can effectively and accurately locate ground faults in distribution networks with offsets.

(2) Since the methodology is immune to the effects of load changes and the necessary measurements are very convenient, it can be widely used for fault location in power distribution networks.

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