

## CHAOS IN POWER TRANSFORMERS INCLUDING THE CORE HYSTERESIS

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### ABSTRACT

This paper presents the study of chaos in power transformers, including hysteresis effect in the core. In previous studies, [1,2,3], the hysteresis effect was not considered and the magnetic characteristics of the core were represented by polynomials. Since hysteresis causes loss of energy in a transformer and usually is represented by a resistor in the equivalent circuit, it seems to have a damping effect against any oscillation in the system. Furthermore, hysteresis plays more complicated role in system behavior by increasing non-linearity of the system. This can be illustrated by comparison between the simulation results of both cases (with and without hysteresis ) considering various system parameters.

**KEY WORDS:** Chaos, Power Transformer, Hysteresis

### 1. Introduction

A Canadian research group [1,2,3] has studied chaos in power transformers in 1993,95,97 respectively, neglecting core hysteresis. The system model consisted of a power transformer, connected to a transmission line, with one phase open. Transformer was assumed unloaded. This condition tends transformer to resonant along the coupled transmission line capacitors and the source voltage. This resonance and oscillations may become chaotic in some conditions.

In this study, however, we attempt to find the impact of core hysteresis on the system behavior. It also clarifies that, state trajectories, moves along new orbits, when hysteresis is modeled accurately instead of presenting system losses by a simple parallel resistor. New attractors which we choose to call hysteresis traps, were also probed.

### 2. System description

The circuit model is shown in Fig. 1. A three-phase power transformer is connected to the transmission line

with one unloaded open phase.  $C_m$  is line to line and  $C_g$  is line to ground capacitances.  $C_m2$ ,  $C_g1$  and  $C_g2$  will be charged directly by sources  $V1$  and  $V2$  can be omitted,

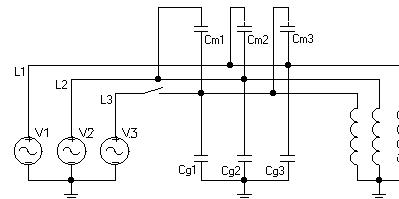
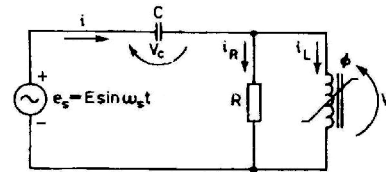


Figure 1: Circuit model

then model can be simplified to circuit in Figure 2. By applying Thevenin law. The equivalent series capacitance and source voltage are shown in (1):



$$C = C_g + 2C_m \text{ and } E = V \frac{C_m}{C_g + 2C_m} \quad (1)$$

Fig. 2: Simplified model

Resistor  $R$  models transformer losses. Neglecting hysteresis, the  $\Phi$ - $I$  characteristic of the transformer has been simulated by the eleventh order polynomial given in equation 2. Curves in figure 3 show the  $\Phi$ - $I$  curves, with different polynomial orders ( $n$ ) :  
 a: $n=5$ , b: $n=7$ , c: $n=9$ , d: $n=11$ , e: $n=15$ .

$$i = a\phi + b\phi^{11} \quad (2)$$

Although, hysteresis effect is not included in this

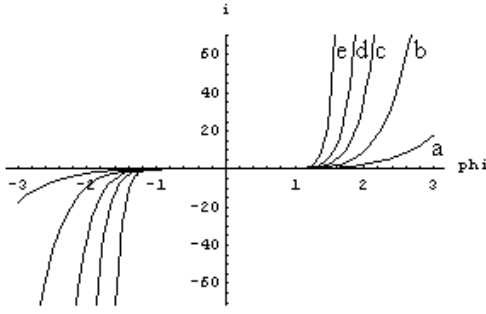


Fig.3: Core Saturation curves for different polynomial orders (n)

model, the saturation effect is fairly good for transient analysis, choosing n greater than 7. Other polynomials with higher orders have been used [4,9] to simulate saturation but no considerable advantage can be seen in the results.

The second order differential equation of simplified system is shown in (3), with nonlinear term corresponding to the iron core of transformer, in which, n is substituted by 11.

$$\frac{d^2\Phi}{dt^2} + \frac{1}{RC} \frac{d\Phi}{dt} + \frac{1}{C} (\alpha\Phi + b\Phi^{11}) = \omega_s E \cos(\omega_s t) \quad (3)$$

Equation (3) is the base of the most of simulations in the study of chaotic behavior of power transformers in which the core non-linearity is included [1,2,3,9].

### 3. Hysteresis model

Several models have been presented for hysteresis phenomenon, with different degrees of accuracy and complexity [4,5,6,7,8,10]. The model presented in [4] with some modifications has been selected for this study.

In this model, the basic saturation curve is defined by a polynomial as shown in Equation (4):

$$i_b = a\phi + b\phi^{n1} + c\phi^{n2} \quad (4)$$

In this study, (n1=11) and (c=0) have been chosen. Now, It is assumed that there exists a hysteresis-related component of current, proportional to the change in  $\Phi$ , from the previous reversal point (Equation 5):

$$i_h = K_h (\phi - \phi_{rev}) \quad (5)$$

Therefore, i, is the sum of the two components,  $i_b$  (base current) and  $i_h$  (hysteresis current), shown in Equation. (6).

The resultant hysteresis loops are shown in Fig. 4.

$$i = i_b + i_h \quad (6)$$

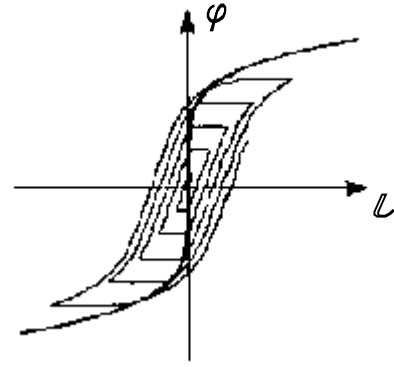


Fig. 4: Hysteresis loops plotted based on the Equation (6)

To improve the model,  $i_h$  can be represented by a curve instead of a line, for example by using a second order polynomial for  $K_h$  as follows in Equation (7):

$$K_h = K_{h0} + K_{hq} \phi^q \quad (7)$$

$$i_h = (K_{h0} + K_{hq} \phi^q) (\phi - \phi_{rev}) \quad (8)$$

Therefore  $i_h$  can be modified by equation (8).

And the total magnetizing curve is represented by equation (9):

$$i = i_b + i_h = a\phi + b\phi^{n1} + c\phi^{n2} + (K_{h0} + K_{hq} \phi^q) (\phi - \phi_{rev}) \quad (9)$$

In this study, the following values have been chosen to simulate core hysteresis curve [9]:

a=2.8e-3	b=7.2e-3
c=0	n1=11
n2=0	Kh0=0.05
q=2	Khq=0.02

### 4. Sensitivity analysis and study of chaos (with and without hysteresis effect)

According to the following sensitivity analysis, it is obvious that some parameters have vital roles in taking the system into the chaotic region. Initial condition, system losses, input voltage amplitude are three parameters, which have the most sensitive roles in system chaotic behavior.

#### 4a. System losses

System losses, normally are modeled by a resistor R. For loss-less system, the parameter R is set to  $R=1e20$ , also  $C=777\text{nf}$  is selected, which corresponds to a typical line with 100km length. An equivalent voltage (An equivalent voltage on a transformer terminals) is choused to be,  $E=0.15\text{pu}$ . Figures 5 and 6 show the simulation results:

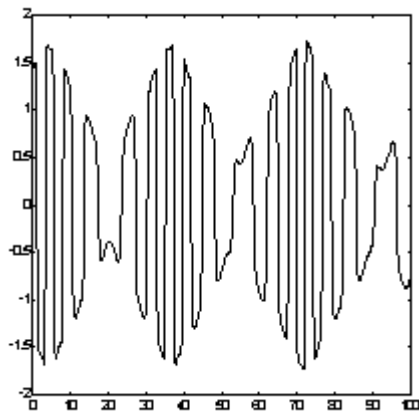


Fig. 5: winding voltage ( VL)

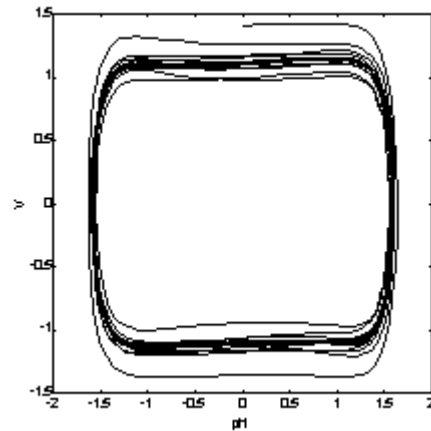


Fig 8: Phase plain map for R=25K (iL/Vc)

The critical value for R has been found 20k ohms for which, Figures 9 and 10 show related simulation results for winding voltage and phase plain map.

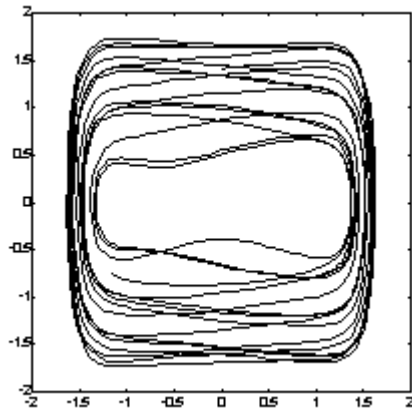


Fig.6: Phase plain Map (iL/Vc) for loss-less system

If R is reduced from 1e20 to 25k, voltage oscillation still exists, but becomes more smooth as in figures 7 and 8:

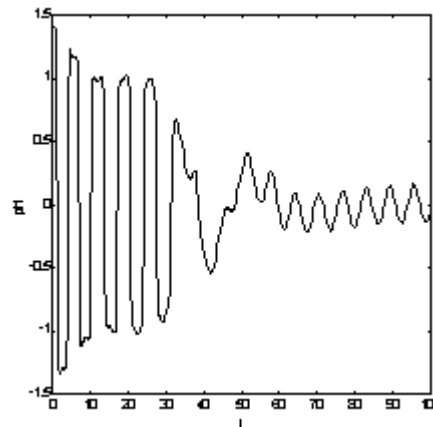


Fig 9: winding voltage for R=20k (VL)

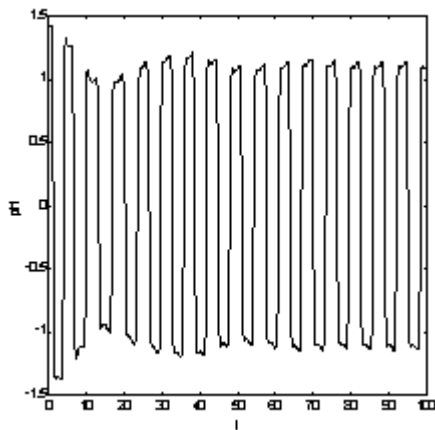


Fig 7: winding voltage (VL)

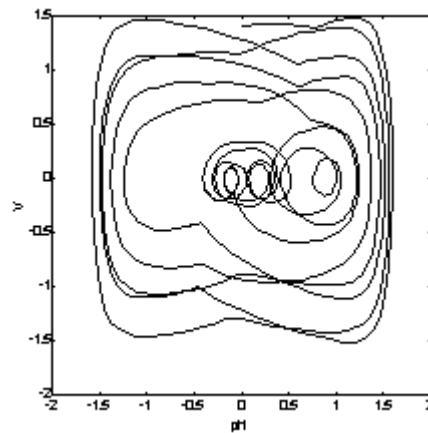


Fig 10: phase plain map for R=20k (iL/Vc)

By using hysteresis model instead of normal saturation curve, new results will be obtained as shown in Figures 11, 12 and 13.

As seen in these figures, the system states will be attracted to the region with lower amplitude and more stability. The improved hysteresis model has been used for this analysis and hysteresis area checked to be covered by system states in a long run. Figure 13, shows how phase plane map is being formed along the time axis and will give a good idea for the system states, when they change by the passage of the time.

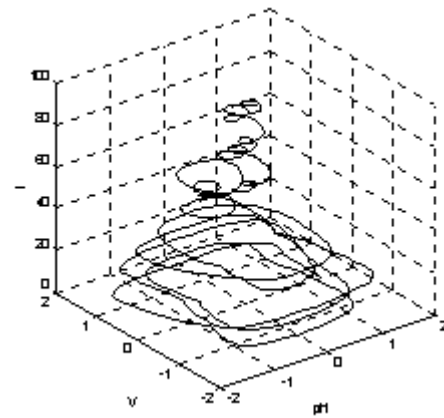


Fig13: Formation of a phase plain

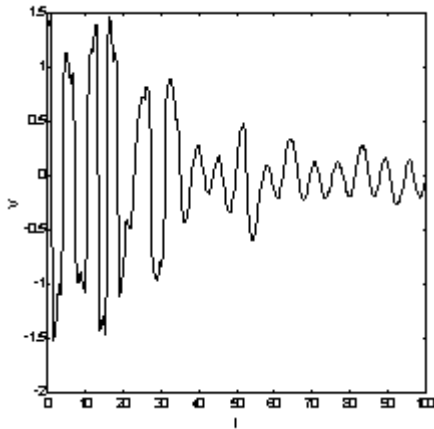


Fig. 11: Winding voltage when hysteresis effect is included

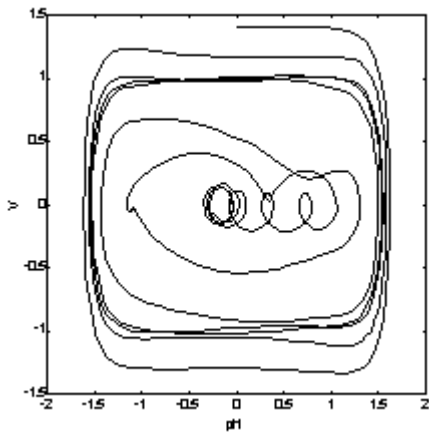


Fig. 12: phase plain map for a system when hysteresis effect is included

#### 4b. Equivalent input voltage

When voltage is doubled, hysteresis will dominate the system states values instead of putting them in the worst chaotic points. Fig.14 and 15, show the results, when system is energized by double normal equivalent voltage and Fig.16, 17 and 18, shows the same situation, when hysteresis exists. Hysteresis apparently damps chaotic oscillation amplitudes to the safe values, but higher frequencies appear due to the additional non-linearity, which is due to the hysteresis phenomenon.

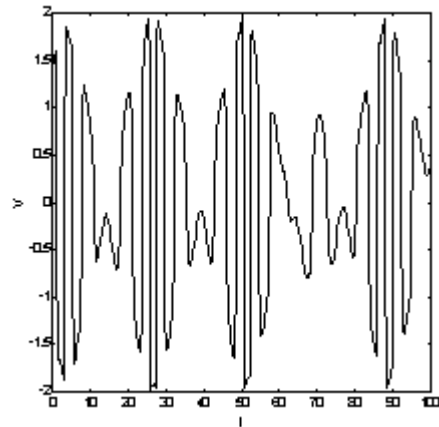


Fig.14: Winding voltage when E=0.3pu

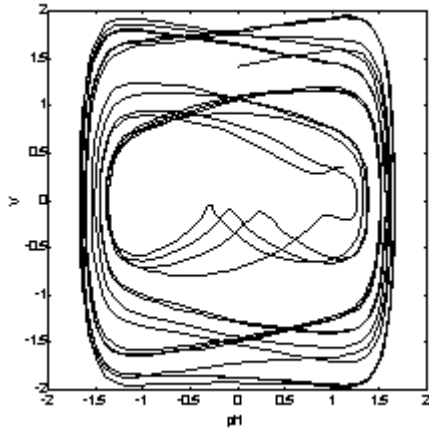


Fig.15: Phase plain map, when  $E=0.3Pu$

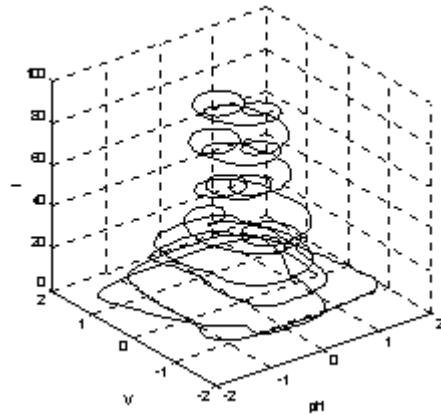


Fig.18: Formation of a phase plain, when hysteresis effect is included

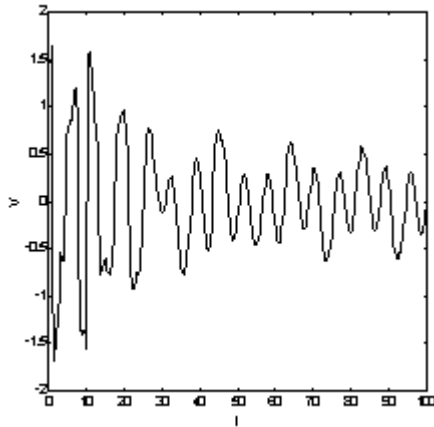


Fig.16: Winding voltage when hysteresis effect is included

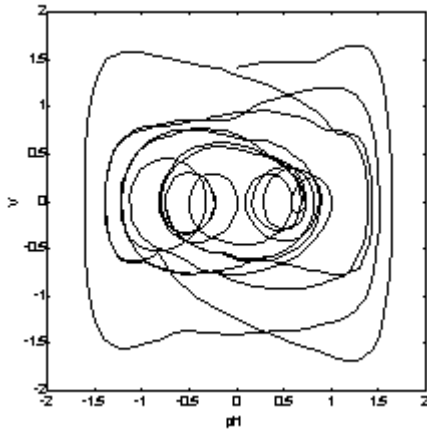


Fig.17: Phase plain when hysteresis effect is included

#### 4c. Length of transmission line

Changing the length of a transmission line will cause change in line to line ( $C_m$ ) and line to ground ( $C_g$ ) capacitance (see figure 1) and therefore equivalent capacitance ( $C$  in figure 2). This parameter was seen to be less sensitive parameter in a system chaotic behavior, related to other parameters ( $E$  and  $R$ ).

Figures 19 and 20 show the results, when the line length is tripled. If transmission line length extends to six times of initial value, it can be seen that the voltage amplitude remains at reasonable value, but becomes smoother. Figures 21 and 22 show the results. To create major changes in the system,  $C$  should change greatly, which is not physically and practically true.

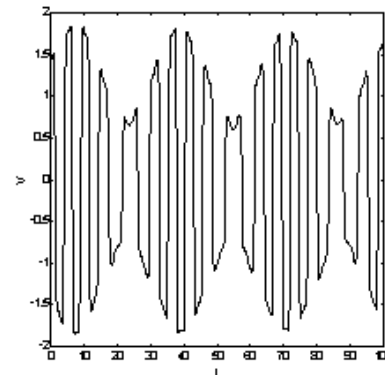


Fig.19: winding voltage when length of line is tripled

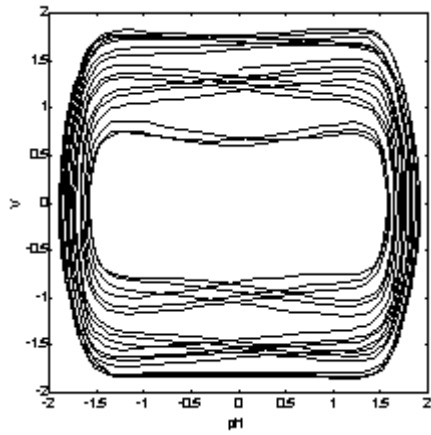


Fig20: Phase plain Map (  $iL/Vc$  )

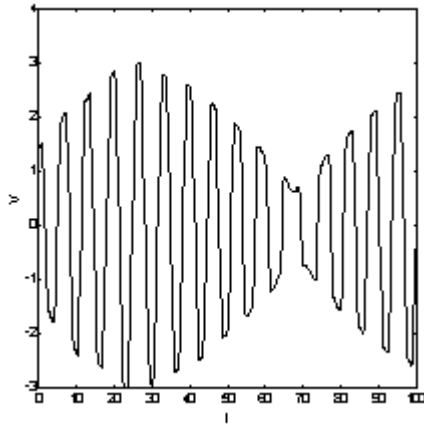


Fig.21: winding voltage when length of line is 6 times of the basic value

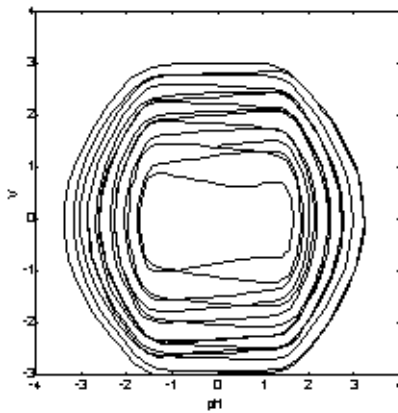


Fig.22: Phase plain Map for line with 6 times longer than the basic value. (  $iL/Vc$  )

#### 4d. Hysteresis parameters

The effect of hysteresis loop width is also important. There is a critical width of hysteresis loop that can put system states in some special orbits, which we chose to call them *chaotic hysteresis traps*. Figures 23 to 27 show

simulation results when hysteresis width exceeded the critical value. This means that hysteresis may put system states ( ie: Voltage and flux ) in some undesired states and trap them in special orbits. In this situation damping of oscillations is the same as when losses modeled by simple resistor however, trajectories are placed in different orbits.

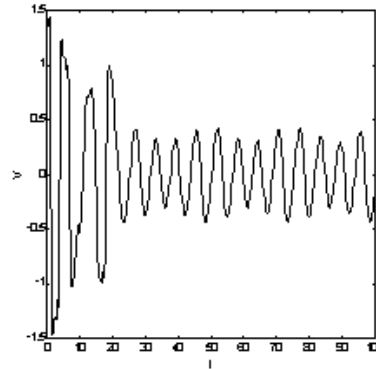


Fig.23: winding voltage when hysteresis width exceeds the critical value.

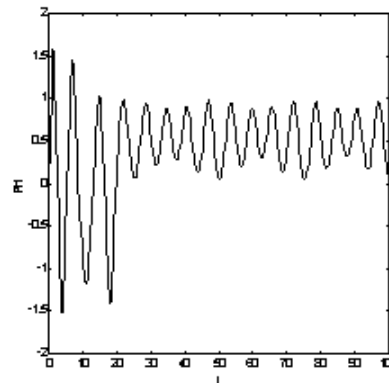


Fig.24: Core Flux, biased on a DC level due to the hysteresis trap

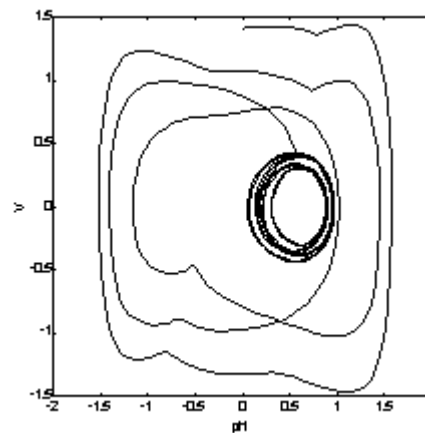


Fig.25: Phase plane Map & hysteresis trap

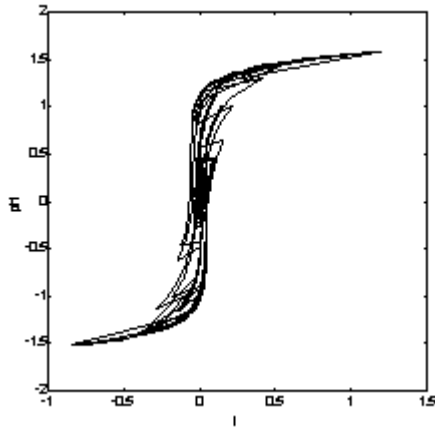


Fig.26: Flux trajectories in hysteresis map

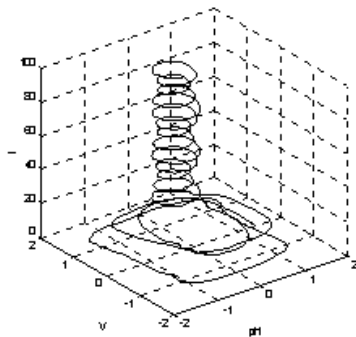


Fig.27: Formation of phase plane diagram ( $iL/Vc/t$ )

## 5. Conclusion

This paper analyses bifurcation and chaos in power transformer, including hysteresis effect in the core. Results show generally that hysteresis acts like other losses in transformers and damps oscillations amplitude, but phase plane or poincare maps show that the state trajectories move along more complicated tracks and have different basin. Besides, system trajectories may fall and be trapped into the special orbits, which we can choose to call **chaotic hysteresis traps**, which may completely change state regions. This phenomenon can be obviously seen by comparing figures 25 and 10. This study shows that in studying the chaos in transformers the core hysteresis cannot be neglected. This is due to the fact that system states as well as their basin and changes can move to new region.

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