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COMPARATIVE STUDY BETWEEN A THREE AND FIVE LEVEL WRPWM SWITCHING SCHEME

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ABSTRACT
Multilevel Random Pulse Width Modulation (RPWM) schemes have drawn increasing attention from researchers in the past few years. The combination of multilevel topologies and random PWM schemes lead to high quality output waveforms and a reduction in discrete harmonics spectra. Research findings on 2-, and 3-level weighted random PWM schemes have been documented in the literature [1]-[3]. This paper presents a comparative study between the performances of a 3- and 5-level WRPWM scheme. The effects of using an even number of comparisons on the performance of both 3- and 5- level WRPWM schemes are discussed.

KEY WORDS
Multilevel Inverter, Weighted Random Pulse Width Modulation, Discrete Harmonic Spectrum, Signal and Noise Power.

1. INTRODUCTION
WEIGHTED random pulse width modulation has been recently proposed for 2- and 3-level inverters [1]-[3]. Predicted and experimental results show that a WRPWM scheme has better control on the resulting frequency spectra of the RPWM signal compared to the standard random switching method. Basically, a WRPWM scheme makes several comparisons between the random number and sinusoidal reference within each sampling period. The majority decision can then be used to determine the switching state of the RPWM output. If the weighted decision process is applied to the entire fundamental period, the probability of having a rectangular voltage block in the central region of each half cycle of fundamental component is high because the weighed switching decision makes it more likely that the switching state is +1 (-1) in the positive (negative) half cycle in the case of 2- and 3-level schemes. Thus the quality of the PWM output waveform is less dependent on the quality of the random number generator and the sampling frequency [1].

The multilevel inverter topology leads to switch stress and loss reduction [4]. Several topologies have been given serious consideration by industry. Among them the best known are the H-bridge cascade inverter, the capacitor clamping inverter [5,6] and the diode clamping inverter [5,7].

The combination of multi-level topologies with various RPWM has drawn interest from researchers in the last few years. The WRPWM method originally developed for 2-level inverters has been extended to 3-level inverters [3]. It was shown that the 3-level WRPWM scheme yields a better spectral performance than either the 2-level RPWM or the 2-level WRPWM scheme [3]. In this paper, a detailed comparison between the performances of a 3- and 5-level WRPWM switching scheme is presented. Both theoretical predictions and experimental results are utilized to carry out the comparison. It is shown that for both 3- and 5-level WRPWM schemes, operating with an even number of comparisons leads to significantly higher fundamental component and signal power but at the expense of increased discrete and continuous noise power. WRPWM schemes that tend to select the zero-level of the DC-bus regularly generate low continuous noise power. Additionally, 3-level schemes operate with lower discrete noise power and average switching frequency but higher continuous noise power compared with 5-level schemes.

2. ANALYSIS OF MULTILEVEL WRPWM METHODS

2.1 General Function of 3- and 5-level WRPWM Schemes
The outputs of 3- and 5-level WRPWM inverters may assume any one of the following values: $+V_d/2$, $+(((l-1)/2)-1)V_d/(l-1)$, $+(((l-1)/2)-2)V_d/(l-1)$, 0, …… $-(((l-1)/2)-2)V_d/(l-1)$, $-(((l-1)/2)-1)V_d/(l-1)$ and $-V_d/2$, where l=3 or 5 and represents the number of levels. $+V_d/2$ and $-V_d/2$ are the DC-bus rail voltages referenced to the center tap point of the dc voltage source. The output of the 5-level inverter, at any one time interval $[nT, (n+1)\,T]$ is determined using the following process.

- Generate random numbers $R_1, \ldots, R_N$ (where N is an integer) with a uniform distribution in the range [0, 1].
- Compare the random number with a sampled reference signal $r(t)$, which is a sinusoidal

waveform. Let c be the number of times that these random numbers are smaller than or equal to $r(t)$. The sinusoidal reference $r(t)=0.5[1+m_a \sin(2\pi f_r t)]$, where m_a is the modulation index, f_r is the frequency of the sinusoidal waveform, and t is the time variable.

- For an l-level inverter, the number of comparisons in one sampling period has to be at least l (i.e., $N \geq l$). This is in order for the system to have enough comparisons to decide which level of dc-bus the switches should connect to.

The 3-level WRPWM signal k(t) was derived in [3]. An equivalent signal for a 5-level scheme is

$$k(t)=\begin{cases} 2A, & \text{if } \left\lceil \frac{N}{2} \right\rceil + q \leq c \leq N, \\ A, & \text{if } \left\lceil \frac{N}{2} \right\rceil + 1 \leq c \leq \left\lceil \frac{N}{2} \right\rceil + q - 1, \\ 0, & \text{otherwise} \left(\left\lfloor \frac{N}{2} \right\rfloor \leq c \leq \left\lceil \frac{N}{2} \right\rceil \right), \\ -A, & \text{if } \left\lfloor \frac{N}{2} \right\rfloor - q + 1 \leq c \leq \left\lfloor \frac{N}{2} \right\rfloor - 1, \\ -2A, & \text{if } \quad 0 \leq c \leq \left\lfloor \frac{N}{2} \right\rfloor - q, \end{cases} \tag{1}$$

where N represents number of comparisons in each sampling interval and q is a choice of design which is an integer between $\left\lfloor \frac{l}{2} \right\rfloor$ and $\left\lceil \frac{N}{2} \right\rceil$.

2.2 Distribution functions of 3- and 5-level WRPWM Schemes

For a 3-level WRPWM scheme, it was shown that three distribution functions are of interest [3]. In the case of a 5-level WRPWM scheme five distribution functions are of interest. By applying the binomial probability law [8] the five distribution functions can be expressed in a form similar to that for the 3-level functions in [3] as follows:

$$F_{N,q}^{(5)}(x)=\begin{cases} 1 & \text{for } x>1 \\ \displaystyle\sum_{m=\left\lceil \frac{N}{2} \right\rceil + q}^{N} \binom{N}{m} x^m (1-x)^{N-m} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x<0 \end{cases} \tag{2a}$$

$$F_{N,q}^{(4)}(x)=\begin{cases} 0 & \text{for } x>1 \\ \displaystyle\sum_{m=\left\lceil \frac{N}{2} \right\rceil + 1}^{\left\lceil \frac{N}{2} \right\rceil + q - 1} \binom{N}{m} x^m (1-x)^{N-m} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x<0 \end{cases} \tag{2b}$$

$$F_{N,q}^{(3)}(x)=\begin{cases} 0 & \text{for } x>1 \\ \displaystyle\sum_{m=\left\lfloor \frac{N}{2} \right\rfloor}^{\left\lceil \frac{N}{2} \right\rceil} \binom{N}{m} x^m (1-x)^{N-m} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x<0 \end{cases} \tag{2c}$$

$$F_{N,q}^{(2)}(x)=\begin{cases} 0 & \text{for } x>1 \\ \displaystyle\sum_{m=\left\lfloor \frac{N}{2} \right\rfloor - q + 1}^{\left\lfloor \frac{N}{2} \right\rfloor - 1} \binom{N}{m} x^m (1-x)^{N-m} & \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x<0 \end{cases} \tag{2d}$$

$$F_{N,q}^{(1)}(x)=\begin{cases} 0 & \text{for } x>1 \\ \displaystyle\sum_{m=0}^{\left\lfloor \frac{N}{2} \right\rfloor - q} \binom{N}{m} x^m (1-x)^{N-m} & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } x<0 \end{cases} \tag{2e}$$

Typical plots of the five distribution functions for a 5-level scheme operating with N=6 and N=7 are shown in Fig.1. Similar plots for a 3-level scheme were presented in [3].

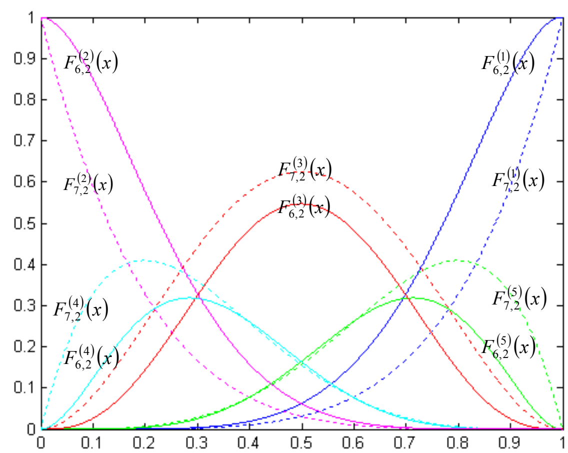


Fig.1: Distribution functions for N=6 (bold) and N=7 (dashed).

From Fig.1, it can be seen that the 5-level WRPWM scheme operating with N=6 is more likely to connect the output terminals to $\pm V_d/2$ but less likely to connect the output terminals to $\pm V_d/4$ or the zero voltage level of the DC-bus than the scheme operating with N=7. This suggests that the scheme operating with N=6 will generate a larger fundamental component than the scheme with N=7 for a given m_a. In [3], it was shown that the 3-level WRPWM scheme generates lower continuous noise than the 2-level scheme because the former has the option of selecting a zero voltage level something that the 2-level scheme is not able to do. Hence one would expect the 5-level scheme operating with N=7 to generate less continuous noise than the scheme with N=6.

2.3 Expected Value of 3- and 5-level WRPWM Signals

The expected value of a random signal is defined [9] as

$$E\{k(t)\} = \sum_{l=1}^{5} k_l P_r \{k(t) = k_l | r(nt) = x\}, \quad (3)$$

where k is the outcome of the random signal (i.e., $V_d/2$, $V_d/4$, 0, - $V_d/4$ or - $V_d/2$) and l is the number of levels. From (3) the expected value of the random WRPWM output function $E\{k(t)\}$ for a 5-level is

$$\begin{aligned} E\{k(t)\} &= P_r\{k(t)=2A\}(2A) + P_r\{k(t)=A\}(A) + P_r\{k(t)=0\}(0) \\ &\quad + P_r\{k(t)=-A\}(-A) + P_r\{k(t)=-2A\}(-2A) \\ &= 2AF_{N,q}^{(1)}(x) + AF_{N,q}^{(2)}(x) - AF_{N,q}^{(4)}(x) - 2AF_{N,q}^{(5)}(x) = g(t) \end{aligned} \quad (4)$$

2.4 Expected Power Spectral Density

With random signals or processes, autocorrelation function is a more appropriate measure for the average rate of change of a random process [10]. The Fourier transform of an autocorrelation function of a random signal gives the expected power spectral density of that random signal. The expected power spectral density in turn gives the average power distribution of a random signal at different frequencies. The expected power spectral density [9] [10] is

$$S(f) = \Im[R_k(\tau)] = \Im\left[\lim_{W\to\infty} \frac{1}{2W}\int_{-W}^{W} E\{k(t)k(t+\tau)\}dt\right] \quad (5)$$

To evaluate the time-averaged autocorrelation, $R_k(\tau)$, it is necessary to determine $E\{k(t_1)\ k(t_2)\}$. If the continuous variable x is replaced by the discrete sampled value of the modulating function $r(\lfloor t/T \rfloor T)$, two cases need to be considered. The first one is when t_1 and t_2 are in the same sampling period. With reference to [2], [3] an expression for $E\{k(t_1)\ k(t_2)\}$ for a 5-level WRPWM scheme is

$$\begin{aligned} E\{k(t_1)k(t_2)\} &= 4A^2F_{N,q}^{(1)}\left(r\left(\left\lfloor \frac{t_1}{T}\right\rfloor T\right)\right) + A^2F_{N,q}^{(2)}\left(r\left(\left\lfloor \frac{t_1}{T}\right\rfloor T\right)\right) \\ &\quad + A^2F_{N,q}^{(4)}\left(r\left(\left\lfloor \frac{t_1}{T}\right\rfloor T\right)\right) + 4A^2F_{N,q}^{(5)}\left(r\left(\left\lfloor \frac{t_1}{T}\right\rfloor T\right)\right) \\ &= U_1 \end{aligned} \quad (6)$$

This represents the instantaneous expected power of k(t) since the average power content of a signal is [9], [11]

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} E\{k^2(t)\}dt \quad (7)$$

The second case that needs to be considered is when t_1 and t_2 are in different sampling periods. Again, with reference to [2], [3] an expression for $E\{k(t_1)\ k(t_2)\}$ for a 5-level inverter is

$$E\{k(t_1)k(t_2)\} = g(t_1)g(t_2) = U_2. \quad (8)$$

where a general expression for g(t) is

$$g(t) = E\{k(t)\} = A\begin{pmatrix} 2F_{N,q}^{(1)}\left(r\left(\left\lfloor \frac{t}{T}\right\rfloor T\right)\right) + F_{N,q}^{(2)}\left(r\left(\left\lfloor \frac{t}{T}\right\rfloor T\right)\right) \\ - F_{N,q}^{(4)}\left(r\left(\left\lfloor \frac{t}{T}\right\rfloor T\right)\right) - 2F_{N,q}^{(5)}\left(r\left(\left\lfloor \frac{t}{T}\right\rfloor T\right)\right) \end{pmatrix}. \quad (9)$$

The Kronecker δ function [8] is used to obtain a general expression for E{k(t_1) k(t_2)} that is valid for the preceding two cases. With reference to (6) and (8), this expression is

$$E\{k(t_1)k(t_2)\} = g(t_1)g(t_2) + d(t_1)\delta_{\lfloor t_1/T\rfloor,\lfloor t_2/T\rfloor} \quad (10)$$

where

$$d(t) = U_1 - (g(t))^2 \quad (11)$$

From (11) it can be seen that $d(t)$ is equal to the variance of the random signal [8] (i.e., $E\{k(t_1)\ k(t_2)\}$- $m_k(t_1)m_k(t_2)$ where $t_1 = t_2$). It is known that by varying the switching patterns the discrete spectral components can be transformed into continuous components. Thus, $d(t)$ represents the instantaneous continuous noise of the 5-level WRPWM signal. Since the expression $E\{k(t_1)\ k(t_2)\}$ is the sum of two terms, so is $R_k(\tau)$. Thus the autocorrelation function can be expressed as

$$R_k(\tau) = R_1(\tau) + R_2(\tau) \quad (12)$$

For the first term $R_1(\tau)$, it is evident that $g(t)$ is periodic, with period ($1/f_r$) if α is an integer. Then a Fourier series expansion for $g(t)$ exists. With reference to [13] it can be shown that

$$R_1(\tau) = \sum_{n=-\infty}^{\infty} |c_n|^2 e^{j2\pi nf_r\tau} \quad (13)$$

where c_n are the Fourier series coefficients of the periodic signal $g(t)$. The time-average autocorrelation of $g(t)$ is periodic with the same frequency as g(t) and (13) defines the discrete part of the spectrum. For the second term $R_2(\tau)$, $d(t)$ is periodic with period $1/(2f_r)$ if α is an integer and has the average value [2], [3]

$$d_{av} = 2f_r\int_0^{1/2f_r} d(t)dt = 2f_r\int_0^{1/2f_r}\left(U_1 - (g(t))^2\right)dt = \frac{\sum_{n=0}^{\alpha/2-1} d(nT)}{\alpha/2} \quad (14)$$

With reference to [2], [3], [11] an expression for $R_2(\tau)$ is

$$R_2(\tau) = \begin{cases} 0, & \text{for } |\tau| \ge T \\ d_{av}\left(1 - \frac{|\tau|}{T}\right) & \text{for } |\tau| < T \end{cases} \quad (15)$$

It is seen from (15) that $\langle R_2(\tau)\rangle$ is a triangular pulse. By substituting (13) and (15) into (5) and with reference to [11], an expression for the power spectral density $S_k(f)$ is

$$S_k(f) = \Im\left(\sum_{n=-\infty}^{\infty} |c_n|^2 e^{j2\pi nf\tau_r}\right) + \Im\left(\begin{cases} 0, & \text{for } |\tau| \ge T \\ d_{av}\left(1 - \frac{|\tau|}{T}\right) & \text{for } |\tau| < T \end{cases}\right) \quad (16)$$

With reference to [12] and [13] (16) is solved to yield an expression for the power spectral density is

$$S_k(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_r) + Td_{av}\sin c^2(fT) \cdot \quad (17)$$

From (17), the total expected power, $P_{k(t)}$ of the WRPWM signal is

$$P_{k(t)} = \int_{-\infty}^{\infty} S_{k(t)}(f)df = \sum_{n=-\infty}^{\infty} |c_n|^2 + d_{av} \quad (18)$$

From Parseval's theorem [11] the summation term in

(18) may be expressed as

$$\sum_{n=-\infty}^{\infty}\left|c_n\right|^2 = f_r \int_0^{1/f_r}\left|g(t)\right|^2 dt = f_r \int_0^{1/f_r} U_1 dt - d_{av} \qquad (19)$$

An expression for the continuous noise is

$$P_{continuous} = 2\int_0^{2000} S_{continuous}(f)df = 2d_{av}\int_0^{2000/f_{sp}} \sin c^2(y)dy \qquad (20)$$

3. ANALYTICAL AND EXPERIMENTAL RESULTS

Three operations of the 5-level inverter and two for the 3-level inverter are considered here for comparison. Fundamental component, low order harmonics, signal, discrete noise and continuous noise power are examined. Fig. 2 shows the variations of predicted fundamental component $2|c_1|$ with m_a for the both the 3- and 5-level schemes. From Fig. 2 it can be seen that for both the 3- and 5-level WRPWM schemes, the fundamental component becomes larger as the number of comparisons, *N*, increases. Further, the fundamental components from the WRPWM schemes operating with *N* even are larger than those for operation with *N* odd. This is in agreement with the deductions that were reached based on Fig. 1. Additionally, 3-level operation with N=4 has the largest fundamental component followed by 5-level operation with N=6, whereas 5-level operation with N=5 has the smallest fundamental component followed by 3-level operation with N=3.

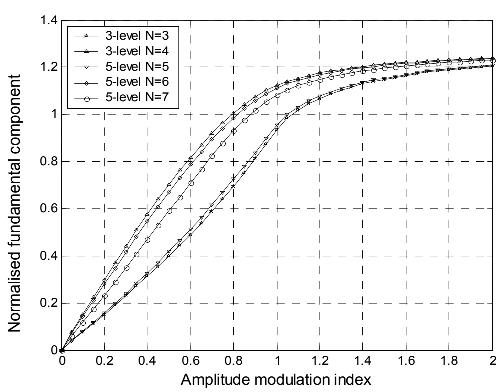


Fig. 2: Predicted fundamental component versus m_a.

Variations of predicted third harmonic with m_a are shown in Fig. 3. It is evident that all the 3- and 5-level WRPWM schemes give rise to third harmonic boosting effect.

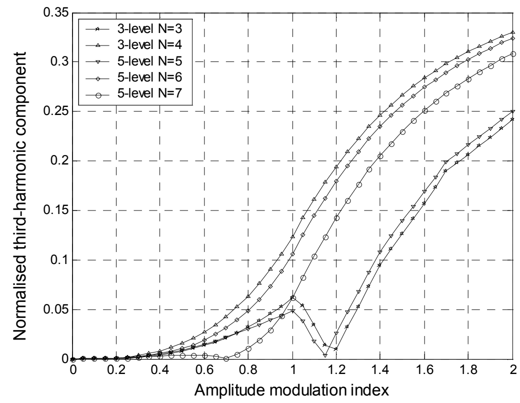


Fig. 3: Predicted third harmonic component versus m_a.

The 3-level scheme operating with N=4 has the largest third-harmonic component followed by the 5-level scheme operating with N=6 and the characteristics of the two operations are similar in shape. In general, the third harmonic increases with m_a and the number of comparisons, N. Moreover, 3- and 5-level WRPWM schemes operating with N even generate a higher third harmonic than those with odd numbers of *N*.

The variations of predicted fifth harmonic $2|c_5|$ with m_a for both the 3- and 5-level schemes are shown in Fig. 4. It is seen that in the region m_a<1, only the 5-level scheme with N=7 has a fifth harmonic component exceeding 0.02 p.u.

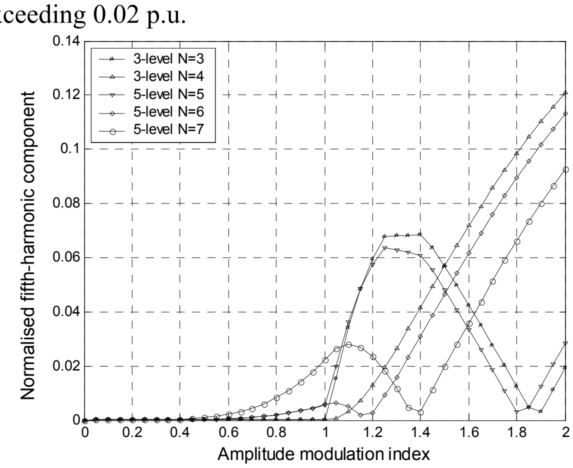
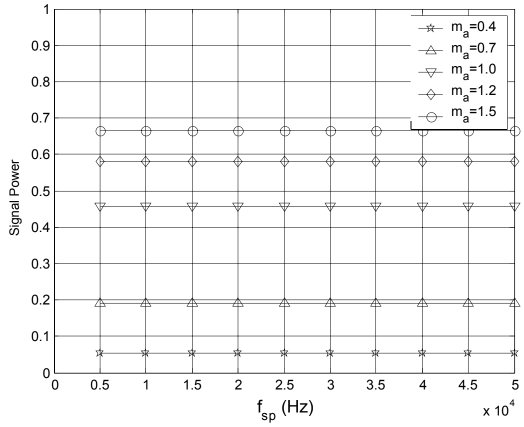


Fig. 4: Predicted fifth harmonic component versus m_a.

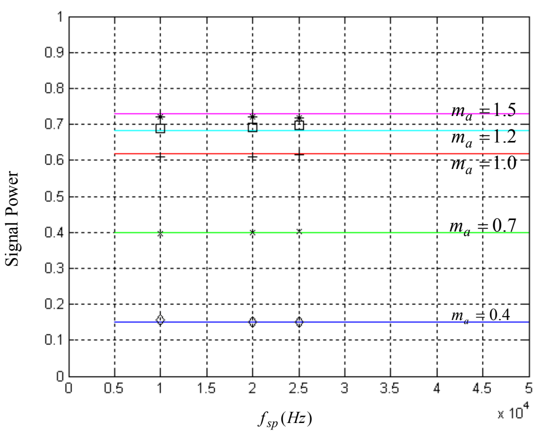
In the over-modulation region, the characteristic for the 3-level scheme operating with N=3 is similar to that due to the 5-level scheme operating with N=5. Both increase with increase in m_a, attain a maximum value (not exceeding 0.07 p.u.) after which they decrease to almost zero before increasing again. This is unlike those for the 3-level scheme with N=4 and 5-level scheme with N=6 where the amplitude of the fifth harmonic always increases with increase in m_a.

Figs. 5-9 present the variations of signal, discrete noise and continuous noise power with f_{sp}. These are for the 3-level with N=4 and the 5-level with N=5 and 6. It is seen from Figs. 5-7 that for a given m_a and f_{sp}, the 5-

level WRPWM scheme operating with N=6 generates higher signal, discrete noise and continuous noise power than the 5-level scheme with N=5.



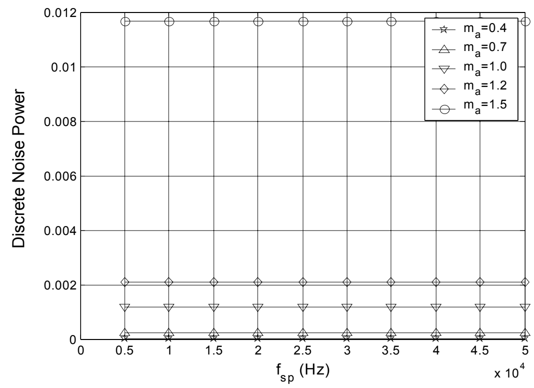
(a) Predicted signal power: 5-level scheme, N=5



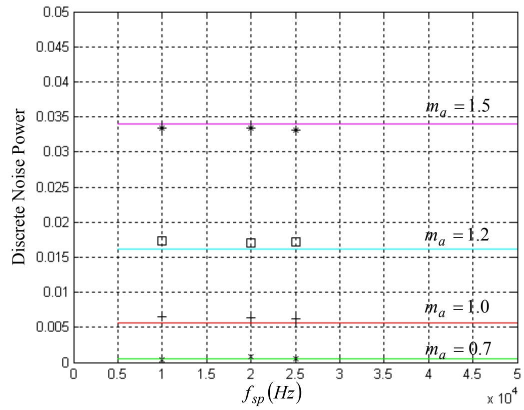
(b) Predicted and experimental signal power: 5-level, $N = 6$.

Fig. 5: Variation of signal power vs f_{sp} for 5-level WRPWM schemes

The lower continuous noise is due to the fact that the 5-level WRPWM schemes operating with an odd number of comparisons are more likely to select the zero voltage level of the DC-bus than those with N even. Schemes operating with an even number of comparisons are on the other hand more likely to select $\pm V_d/2$ voltage levels of the DC-bus thus generating higher fundamental components and signal power.

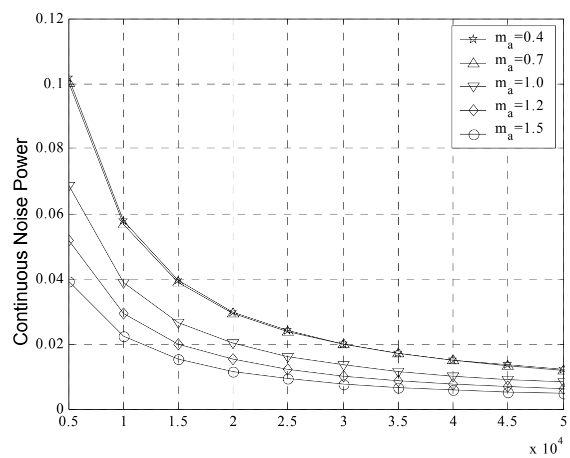


(a) Predicted discrete noise power: 5-level, N=5

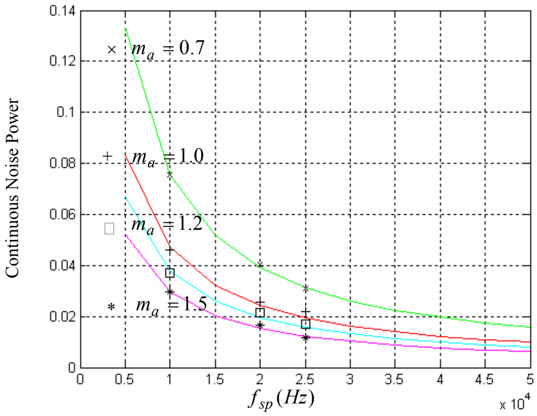


(b) Predicted and experimental discrete noise power: 5-level, $N = 6$

Fig. 6: Variation of discrete noise power versus f_{sp} for 5-level WRPWM schemes



(a) Predicted continuous noise power: 5-level, N = 5



(b) Predicted and experimental continuous noise power: 5-level, $N = 6$

Fig. 7: Variation of continuous noise power versus f_{sp} for 5-level WRPWM schemes

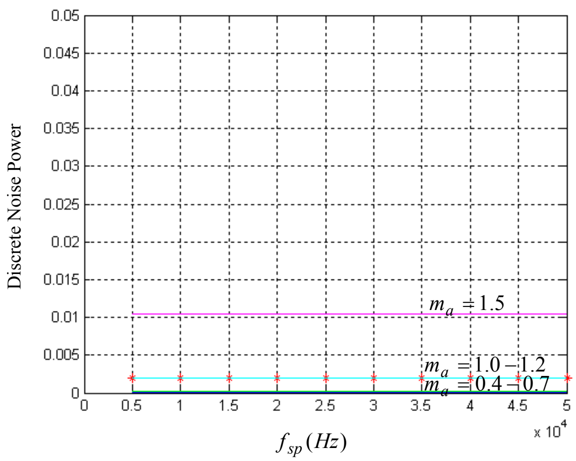


Fig. 8: Predicted discrete noise power versus f_{sp} for 3-level WRPWM scheme with $N = 4$.

From Fig. 8, it is evident that negligible discrete noise generated when m_a=0.4 and 0.7 whereas that for m_a=1.0 and 1.2 is also very low.

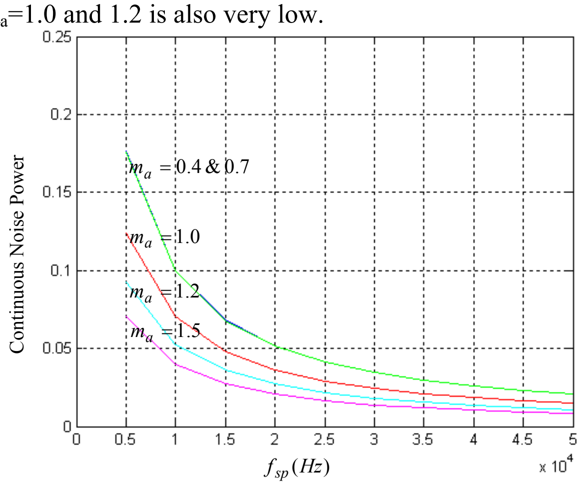


Fig. 9: Predicted discrete noise power versus f_{sp} for 3-level WRPWM scheme with $N = 4$.

Comparing Fig. 6 with 8 shows that the 5-level scheme with N=6 generates higher discrete noise power than the 3-level scheme with N=4 for a given f_{sp} and m_a. Comparing Fig. 7 with 9 shows that the 5-level scheme with N=6 generates lower continuous noise power than the 3-level scheme with N=4 for a given f_{sp} and m_a. Fig. 10 presents variations of the ratio f_{sw}/f_{sp} with m_a for 5-level WRPWM schemes operating with various values of comparisons, N. Similar characteristics were presented in [3] for 2- and 3-level WRPWM schemes.

From Fig. 10, it is seen that for 5-level WRPWM schemes, the highest f_{sw} attained is approximately 0.4 $f_{sp.}$ This is for operation with even number of comparisons and $m_a \approx 0$. When operating with an odd number of comparisons, $f_{sw,max}$ does not exceed $0.35f_{sp}$ and occurs at $0<m_a<1$. In [3] it was shown that the 3-level scheme with N=4 has $f_{sw,\ max}$ not exceeding $0.335f_{sp}$ occurring at $m_a \approx 0$. The 3-level scheme with N=3 has $f_{sw,\ max}$ not exceeding $0.24f_{sp}$. In general, it is seen that the 5-level schemes operate with a higher f_{sw} than the 3-level schemes which in turn have a higher f_{sw} than 2-level schemes for a given f_{sp} and m_a. The difference in f_{sw} is very significant at low values of m_a than at higher values of m_a.

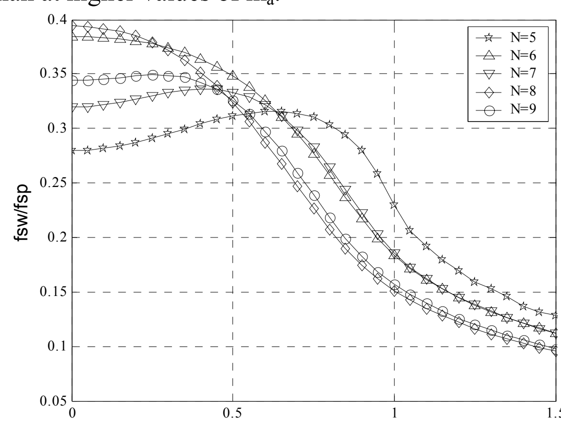


Fig. 10: Predicted f_{sw}/f_{sp} versus m_a: 5-level, N=6, f_{sp}=20 kHz.

A laboratory scale 5-level diode clamped inverter was built to obtain experimental results. A Motorola DSP 86506 was used to implement the 5-level WRPWM schemes. The WRPWM waveforms were monitored with a TDS 220 digital oscilloscope with fast Fourier transform (FFT) facility. The fundamental frequency was 50 Hz. Fig. 11 presents output phase voltage frequency spectrum for a 5-level WRPWM scheme operating with N=5, m_a=1.2 and f_{sp}=20 kHz. Fig. 12 on the other hand presents output phase voltage frequency spectrum for a 5-level WRPWM scheme operating with N=6, f_{sp}=20 kHz and 1.2. From Figs. 11 and 12 it can be seen that 5-level WRPWM scheme employing N=5 generates smaller fundamental and third harmonic components compared with that using N=6. Further, switching frequency harmonics and continuous noise generated is very low.

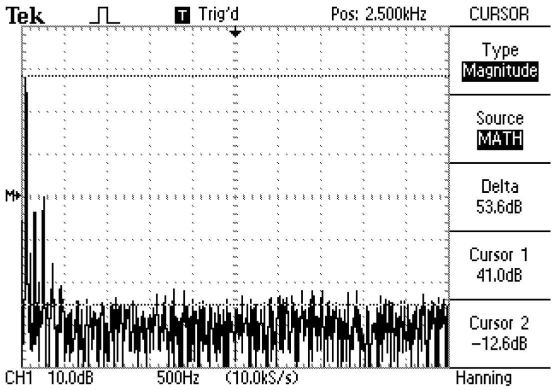


Fig. 11: Output phase voltage frequency spectrum for a 5-level, N=5, m_a=1.2, f_{sp}=20 kHz.

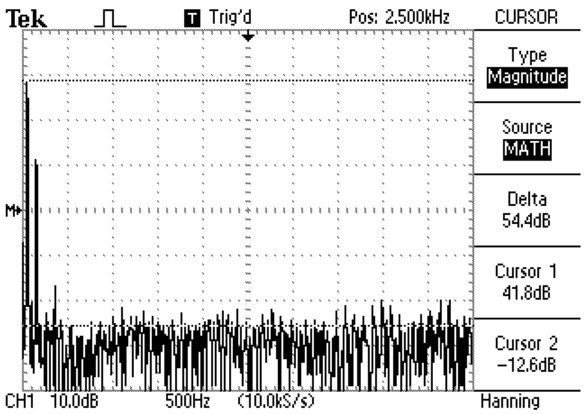


Fig. 12: Output phase voltage frequency spectra for a 5-level WRPWM schemes: N=6, m_a=1.2, f_{sp}=20 kHz.

4. CONCLUSION

A general statistical approach was used to analyse the performance of 5-level WRPWM schemes. A detailed comparison between the performances of 3- and 5-level WRPWM schemes was also presented. It was shown that in general, a WRPWM scheme that has a high probability of selecting the zero voltage level of the DC-bus generates low continuous noise power. This was shown to be true for 5-level scheme operating with N=5 as compared to the scheme operating with N=6. For both the 3- and 5-level WRPWM schemes, operating with an even number of comparisons leads to higher fundamental component, signal power, discrete noise and continuous noise power and average switching frequency than when operating with N odd.

The 3-level WRPWM schemes operate with lower discrete noise power and f_{sw} but higher continuous noise power compared with the 5-level schemes. The analytical and experimental results have shown performance of the 5-level schemes to be comparable to that of the 3-level schemes for a given dc-bus voltage, m_a and f_{sp}. However, the 5-level inverters are capable of working at much higher dc-bus voltages than the 3-level inverters for a given switch blocking voltage. This makes the 5-level WRPWM schemes that much more attractive despite having slightly higher f_{sw} compared with the 3-level schemes.

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