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MODELING OF A PIEZOELECTRIC BEAM FOR VOLTAGE GENERATION

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ABSTRACT
Piezoelectric materials (PZT) have the ability to convert mechanical forces into an electric field in response to the application of mechanical stresses or vice versa. This novel property of the materials has found applications in sensor and actuator technologies. The study presented in this paper targets the modeling of a PZT beam device for voltage generation by transforming ambient vibrations into usable electrical energy. This device can potentially replace the battery that supplies the power in a microwatt range necessary for operating some wireless sensor devices. While the feasibility of this application has been repeatedly demonstrated in the literature, but real working devices are partially successful because of the complex mathematical challenges involved in modeling the operation of such devices. This paper presents a simple theoretical model of voltage generation based on simple classical beam analysis. Experimental investigations were carried out and it was demonstrated that the model was able to predict the voltage generated by PZT on a vibrating beam to within less 22%. The energy harvesting capabilities of the beam device was also experimentally investigated.

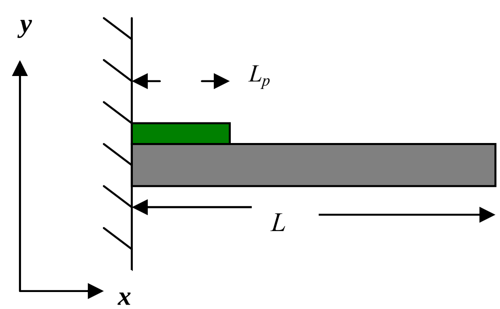
KEY WORDS
Modeling, voltage generation, energy harvesting, and piezoelectric

1. Introduction

Energy harvesting is a form of renewable power generation and is a key enabling technology for a whole host of future distributed systems such as wireless sensors, implantable medical devices, and structural health monitoring. Among the possibilities for energy generators, piezoelectric materials have been generally used because of their ability to couple ambient mechanical energy into electrical energy [2,3]. Recent advancements in the electronics industry have made it possible to reduce the power requirements of most electronic devices to levels comparable to the generation capabilities of piezoelectric harvester devices [1]. Lead zirconate titanate (PZT) is the most used piezoelectric material because its high electomechanical coupling characteristics in single crystals [2-6]. Analytical modeling is an inevitable element in the design process to understand various interrelated parameters and to optimize the key design parameters while studying and implementing such power harvesting devices. There have been some prior researches on the topic of modeling and predicting energy harvesting capability of various structures. Sodano et al [4,5]. Roundy et al [7] presented many research articles on vibration based energy harvesters. The aim of this study was to develop a theoretical model of voltage generation by a piezoelectric beam device and experimentally investigate its validity.

2. Theory and System modeling

Piezoelectric materials physically deform in the presence of an electric field, or conversely, produce an electrical charge when mechanically deformed. This effect is due to the spontaneous separation of charge within certain crystal structures thereby producing an electric dipole. For a piezoelectric material it is known that output voltage of the material is a function of stress. Typically, the stress is achieved through the displacement or bending of the piezoelectric beam due to the vibration.



L_p= length of PZT layer
L=length of substrate beam
w = width of the beam
Figure 1. Cantilever beam made of piezoelectric patch and metal substrate

The schematic of the beam under study is shown in Figure 1. If the thickness of the host substrate layer (y_b) is far greater than the thickness of the PZT patch (t_p); and the length L approximately ten times Lp, then the classical

Euler-Bernoulli theory applies [8]. Modeling is carried out for a case where the forcing function is a harmonic excitation.

For a beam of uniform cross section, the beam curvature is the second derivative of the beam deflection curve:

$$\kappa = \frac{d^2 y}{dx^2} = \frac{M}{EI} \tag{1}$$

where: M is the bending moment, E the Young Modulus of the beam material and I is the cross-sectional moment of inertia.

The average curvature is given by

$$\bar{\kappa}(x) = \frac{1}{L_p}\int_0^{L_p} \frac{d^2 y}{dx^2} dx = \frac{1}{L_p} y'(L_p) \tag{2}$$

The average strain can be described in terms of the average curvature using the relation:

$$S_1(x, y) = -\bar{\kappa}(y - y_c) \tag{3}$$

where y_c is the distance from the neutral axis of the PZT element.

The relation that relates the stress to the strain is $T_1(x, y) = E_p S_1(x, y)$. Using equations (2) and (3) in the relation yields

$$T_1(x, y) = \frac{1}{L_p} E_p (y - y_c) y'(L_p) \tag{4}$$

Choosing the reference to be the interface between the PZT and the substrate, then the magnitude of stress at the surface of the PZT is given by

$$T_1(x, y) = \frac{1}{L_p} E_p t_p y'(L_p) \tag{5}$$

Assuming a static load at the end of the cantilever beam, angular displacement (i.e. slope) at the tip of the beam is [8]:

$$y'(x) = \frac{3a}{2L^3}(2Lx - x^2) \tag{6}$$

where $a = y(L)$ is the displacement at the cantilever beam end.

Evaluating the slope at the end of the PZT element, $x = L_p$ and substituting in equation (5) gives

$$T_1(x, y) = \frac{3a}{2L_p L^3} E_p t_p (2LL_p - L_p^2) \tag{7}$$

The piezoelectric constitutive equation relating electric displacement to applied stress is [9]:

$$D_3(x, y) = \varepsilon_{33}^T E_3 + d_{31} T_1(x, y) \tag{8}$$

where D_3: electric displacement, ε_{33} permittivity of piezoelectric, E_3 electric field strength.

For an open circuit, the dielectric displacement (D) is zero. Thus

$$\varepsilon_{33}^T E_3 = -d_{31} T_1(x, y) \tag{9}$$

From the definition of uniform electric field strength

$$E = \frac{V_{oc}}{t_p} \tag{10}$$

where t_p is the thickness of the piezoelectric material, and V_{oc} is the open circuit voltage

Combining equations (9) and (10) yields

$$V_{oc} = -\frac{t_p d_{31}}{\varepsilon_{33}^T} T_1(x, y) \tag{11}$$

Substituting equation (7) into equation (11) gives the magnitude of the peak output voltage as

$$Voc = \frac{3t_p^2 d_{31} a E_p}{2L_p L^3 \varepsilon_{33}^T}(2LL_p - L_p^2) \tag{12}$$

In this beam model, the forcing function is assumed to be sinusoidal and harmonic. For experimental setups, an electromagnetic shaker is used as the excitation source to drive the beam.

3. Experimental Setup

PZT layer was attached to the aluminum substrate layer using two-part epoxy glue (Fig. 2). PZT model number PSI-5A4E manufactured by Piezo Systems Inc. was used to fabricate the cantilever beam device. The actual clamped length of the Aluminium and the PZT are150 mm and 72.4 mm respectively. The width of the PZT and Aluminium was 25 mm.

Figure 2. PZT attached to aluminium beam

To test the piezoelectric bender, it was clamped on the shaker as shown in Fig. 3. A reflective optical technique that employed a HeNe laser and a small mirror attached to the beam, used to measure the amplitude of the vibrating beam [10].

Figure 3. Experimental Setup

4. Experimental Results and Analysis

A function generator was used to drive the shaker which was used to excite the beam. The first resonance frequency of the beam was measured, and was found to be 58.60Hz.With function generator set at the resonance value, the open circuit voltage generated by the PZT was measured using an oscilloscope (HP 54610B) for different beam vibration amplitudes. The results for the measurements are shown in **Fig. 4.**

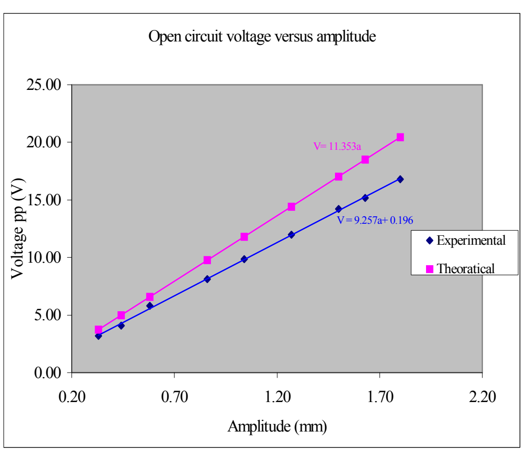


Figure 4. Plot of generated voltage versus vibration amplitude

The developed model predicted a voltage generation of about 11.35V/mm vibration while the experimental investigations showed a voltage generation of 9.26V/mm. Thus, the theoretical prediction of voltage generation was about 22% off compared to the measured, knowing the effects of the bonding layer and the exact damping ratio may reduce the difference.

With the beam device at the first resonance frequency of 58.60 Hz, the voltage generated by the PZT was rectified using a full bridge diode rectifier, and the DC voltage filtered using a 3.3 µF capacitor. The power outputs for different load resistances were obtained (**Fig. 5).**

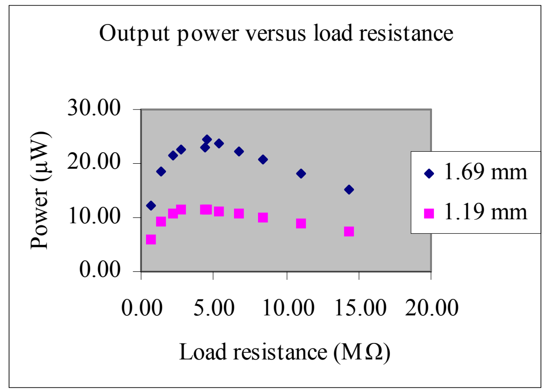


Figure 5. Plot of power curves for 1.69 mm and 1.19 mm for several load resistances

The device was able to deliver up to 25µW to a resistive load of about 4.8MΩ. It is also noted that the power generation performance depends on material properties, shape of the beam structure, and the harvesting circuitry. Accordingly, optimal design of the beam for the given vibration source will also increase the performance.

5. Conclusion

A simple but useful model of voltage generation by a piezoelectric harvesting beam device was successfully developed and was experimentally validated. The power generation capabilities of the fabricated device were investigated for possible application in ultra low power applications. The theoretical and experimental results indicate that ambient vibration offers sufficient power sources for low power systems like wireless sensors. As future research, the baseline findings should be steadily improved through optimization of the design.

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