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A CURRENT DETECTION BASED ON AN EXTENSION OF THE PRONY'S METHOD

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ABSTRACT
Although recent development in harmonic detection methods has made inroads in improving performance and efficiency, all those methods share some significant limitations: the detection system is open-looped and employs a fixed frequency that make it difficult to detect and track; the more frequently used instantaneous reactive power harmonic detection method requires a large number of multipliers, and its vector transformation is rather complex together with many other limitations. The new method presented in this paper reaches a satisfying compromise between accuracy and rapid response time. It incorporates the structure and principle of injective Active Power Filter used for effective harmonic wave management. This new detection method is based on an improved extension of the Prony method for Spectrum Estimation and uses adaptive frequency tracking and current frequency demultiplication. It uses a digital signal processor and a high-speed interfacing device. The method is automated through a simple software program. It provides a real-time detection method with a small error rate which meets the practical needs.

KEY WORDS
harmonic detection; detection accuracy; rapid response; Extension of Prony method for Spectrum Estimation; adaptive;

1. Introduction

To control harmonic wave in power systems, the first step is to provide effective harmonic detection. Recent improvements in modern detection techniques have given rise to the development of many single-phase and three-phase harmonic current detection methods in power systems. Aiming at rapid and accurate detection of harmonic currents or harmonic voltage, these methods ensure the harmonic suppression effect of active power filter, but the detection method of active power filter's compensation current is different from the harmonic measurement method which used in the general power system. The harmonic measurement method doesn't need to break down the various harmonic components generally, but to detect the whole high-order harmonic and reactive current distortion except the fundamental active current, and detect the transient current distortion in the power grid. Accurate and fast detection is one of the key characteristics of an active power filter, and it can provide referent signal for giving precise compensation for the control system of active power filter. It is also one of the keys to determine the performance of the active power filter. Existing harmonic current detection methods are mainly band-stop frequency-selective methods. Fryze time-domain analysis method, the method based on fast Fourier analysis, the method based on instantaneous reactive power theory and the method based on wavelet theory all are the primary means to detect harmonic current in present stage. They obtain reactive information through some certain ways and see it as a reference to control the output of active power filter. Its high speed, accuracy and flexibility have a direct impact on the track of active power filter and the compensation as well as its applications. Although these harmonic detection methods have their own advantages in terms of performance and efficiency respectively, they still have insurmountable problems. One is the open-looped detection system with fixed frequency, which is sensitive to the changes in component parameters and grid frequency; another one is the less-accurate detection without adaptive capacity especially. In that way, it cannot be well-tracked and well-tested. Nowadays, the harmonic detection method based on instantaneous reactive power is widely used and can be achieved through analog circuit. This method performs well in time, but it has some merits in the circuit complexity, the need to adopt a large number of multipliers, the bad adjustment, the large calculation and the complex transform vector. The low-pass filtering algorithm will affect the design of the performance of the harmonic active power filter's compensation performance directly and the heavy dependence on the standard cosine signal (the actual very difficult to produce) also are the insufficiencies of it[4-6]; additionally, existing detection algorithms seldom considerate the actual situation that under the situation which is much greater than harmonic currents, fundamental current will affect current detection; and if the generator output, load and system structure have changed in the distribution network of enterprises, the mutation and frequency of deviations of measured voltage signal in the early phase angle will arise, etc. In this respect, the harmonic current reference signal in accordance with the process of real-time access, accuracy requirements, as well as processor detection algorithm has the need of a small amount of calculation, and the characteristics of easy realization, combining with effective governance applies to the structure and working principle of harmonic injection of active power filter, it makes a compromise between meeting the measurement accuracy and detection performance of the rapid response. This article puts forward a tracking current sub-frequency detection method of adaptive frequency based on improved extension Prony Spectrum Estimation method, which can not only be used to select out the components to be detected in the fundamental frequency of change and mutation in the case of current quickly and accurately, but can also be applied to a rapid test of a single harmonic wave which is very important to the detection algorithm using digital signal processors and simple software program to meet the requirement of small error and high timeliness in practical application.

2. Tracking Current Detection Method based on Improved Extension Prony Spectrum Estimation Method

2.1 Extension Prony Spectrum Estimation method

At present stage, most harmonic detection methods are only able to detect the harmonic in the cycle after the change in the load while the harmonic current detection based on the improved extension of Prony Spectrum Estimation method can make the harmonics detect time less than one cycle [7]. This method is used to approach a sampled data sequence of interval length L with N-index approximation of a combination of amplitude, phase and frequency, and the fundamental component of the active phase is equal to the voltage waveform. Thus, using this method to detect the harmonic current has the following advantages: it only needs the solution of amplitude, its calculation is very small and obtaining the achieving to track changes in the load in less than a fundamental cycle to a certain extent.

According to all above, expressing the actual power system of non-sinusoidal periodic waves as a set of number sequences in discrete interval variables with sampling ^{n}, and approaching load current to the sampled data sequence $\{i_n\}$ of first-class index interval length L with the combination of amplitude B_k, phase θ_k and frequency f_k in N approximations based on the extension Prony Spectrum Estimation method, the approximation is

$$\bar{i}_n = \sum_{k=1}^{m} a_k e^{j\omega_k n\Delta T} \tag{1}$$

n＝0, 1……N-1

In the formula, $\Delta \mathrm{T} = \mathrm{T}/\mathrm{N}$

$$\omega_k = 2\pi f_k$$

$$a_k = B_k e^{j\theta_k}$$

In that, f_k can be arbitrary, and does not require having integer multiple relationship between the fundamental relations, so the method can also detect non-integer harmonics. Because of the actual requirements in accordance with enterprises, the system will not consider that condition, but the f_k referred later should have integer multiple relationships with the fundamental wave.

Expressing the above formula in the matrix form

$$\mathrm{I} = \mathrm{XA} \tag{2}$$

In the formula $\mathrm{I} = \begin{pmatrix} \bar{i}_o & \bar{i}_1 & L & \bar{i}_{N-1} \end{pmatrix}^T$

$\mathrm{A} = (a_0 \quad a_1 \quad \mathrm{L} \quad a_m)^T$

X＝

$$\begin{bmatrix} e^{j\omega_1 0} & e^{j\omega_2 0} & L & e^{j\omega_m 0} \\ e^{j\omega_1 \Delta t} & e^{j\omega_2 \Delta t} & L & e^{j\omega_m \Delta t} \\ M & M & & M \\ e^{j\omega_1 (N-1)\Delta t} & e^{j\omega_2 (N-1)\Delta t} & L & e^{j\omega_m (N-1)\Delta t} \end{bmatrix}$$

This is a problem of nonlinear equations whose solution is difficult. If knowing the power system harmonic current frequency, and that is to say f_k is determined, then X should be changed as a constant coefficient matrix in general occasions, so there is:

$$\mathrm{E} = \mathrm{I} - \mathrm{XA} \tag{3}$$

In which E is a n-dimensional zero vector.

In applications of $\Delta t = 0.02/256 = 78.125\mu s$, discrete signal in the equal time interval is used to replace the measured continuous signals. How much the sampling time interval get on earth has something to do with the system of linear equations related to the redundancy, and the larger the interval is, the fewer the sample points are, which might miss the necessary information; while too small interval and too many sample points will decrease the ratio of signal / error. At the same time, the issue of harmonic frequency resolution is needed to be taken into consideration. Therefore, selecting a moderate value based on experience usually under the condition of satisfying the Nyquist sampling frequency, as long as it meets the condition of N> 2P (P for the effective rank) is the effective strike rank which the number of harmonics is in the know (there is m harmonics), then there is no need to calculate the effective rank P. When P=2m, if not knowing the number of harmonics, it is necessary to take a larger P, and obtain the effective rank of P by using the method of singular value decomposition. This is the problem of harmonic state estimation. However, in the practical applications of active power filter, when compensating all the harmonic frequency is necessary, it is needed to calculate the fundamental harmonics in actual calculations, and then subtract the fundamental component from detected signals; when only a few specific harmonic frequency are needed to compensate, it only needs to calculate the required number of harmonics by the above-mentioned methods. Because in the general three-phase power grid, the main harmonic is 2, 3, 5, 7, 11, 13 times, such 4, 6, 9-harmonic wavelet is non-existent basically and the high-order generally needed to consider harmonic amplitude is very small. So the general m is so small, and doesn't need the entire cycle of signal sampling generally.

Therefore, using this method to calculate harmonic current has small volume of calculation, and its real-time calculation is only the N plural times real multiplication or 2N multiplication, while FFT computation i=s the second complex multiplication of N log$N$. In fact, the information of various harmonic that got from FFT method is not all useful and containing a considerable number of redundant computation. For example, supposing a waveform has a frequency of 50Hz with a period T=0.02S, then setting its sampling number per cycle N= 256, so the period of signal sampling is. $\Delta t = 0.02/256 = 78.125\mu s$ Assuming that the load mutates at T, under the condition of expansion of Prony spectrum estimation method N> 2m, N is smaller than the current number of samples in generally speaking, so it can detect harmonic currents in less than a cycle time.

However, the frequency of voltage will change slowly, and formula (3) is non-common in mathematics. That is to say, it cannot make every part of the vector E zero. If we do not consider this point, the results of harmonic current detecting will certainly have large deviation. In view of this situation, this paper proposes to adopt the LMS algorithm of adaptive adjustment algorithm to optimize the frequency f_k of conductance matrix in extension Prony Spectrum Estimation method online, and obtain the optimal under this condition.

2.2 Improved tracking current detection of extension Prony Spectrum Estimation method

The current frequency detection block diagram of adaptive frequency tracking is as shown in figure 1. In figure 1, $i(n)$ is desired output, $is(n)$ is the result of adaptive adjustment, and $e(n)$ is the error between the desired output and the results of adaptive adjustment. That is:

$$e(n) = i(n) - \overset{S}{i}(n) \tag{4}$$

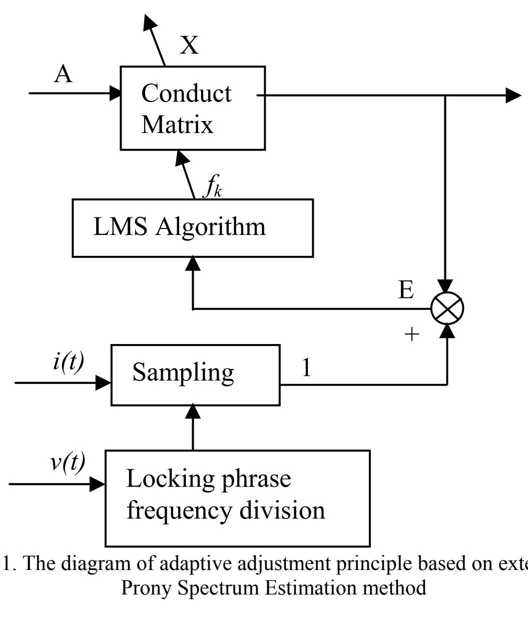

Fig 1. The diagram of adaptive adjustment principle based on extension Prony Spectrum Estimation method

This adaptive algorithm is used to track error signals and adopt LMS algorithm criteria to optimize the calibration frequency voltage online.

From figure 1, and concerning about coefficient optimization, there has

$$i(n) = \sum_{k=1}^{m} x_k(n) a(n) = X_N(n).A_N(n) \tag{5}$$

In this figure,

$X_N(n) = [x_1(n), \quad \cdots, \quad]_{nxm}$

is frequency vector.

$A_N'(n) = [a1(n)a2` \quad ' \quad \cdots, \; a_m(n) \quad]_{nxm}$

is current vector.

From formula (4), there has

$$e^2(n) = [i(n) - \overset{S}{i}(n)]^2 \tag{6}$$

$e^2(n)$ is the square error changed with timing n. Define $\varepsilon(n)$ as the expectations (set average) of $e^2(n)$, that is mean the square error (MSE).

$$\varepsilon(n) = E\{e^2(n)\} \tag{7}$$

Put formula (5), (6) into (7), there is

$$\begin{aligned} \varepsilon(n) &= E\{e^2(n)\} = E\{[i(n) - \hat{i}(n)]^2\} \\ &= E\{[i(n) - X_N A_N(n)]^2\} \\ \varepsilon(n) &= E\{i^2(n)\} - 2X_N E\{i(n) A_N(n)\} \\ &+ X_N E\{A_N(n) A_N'(n)\} X_N' \end{aligned} \tag{8}$$

The adaptive algorithm based on the LMS principle is to obtain a group of $x_k (k = 1, 2 \ldots, m)$, and get the minimum of $\varepsilon(n)$. Through differential set 0 law, that can be got and as well as N equation solutions. Then, there is the result:

$$X_N{}^* = R^{-1}{}_{NN} P_N \tag{9}$$

This is the famous Wiener optimal solution, in which P_N is a time-varying vector and the inter-related volume of $i(n)$, R_{NN} is the autocorrelation matrix of $A(n)$.

Obtaining solutions makes the smallest corresponding X_N of MSE can be got through the method used in calculating formula (9), but problems such as less-accurate calculation comes out when N is too large, and the computation becomes too much together with matrix which has inverse operation. Recursive solution such as steepest gradient method is often used in practical situations:

$$X_N(n+1) = X_N(n) + 2\mu[P_N - R_{NN} A_N(n)] \tag{10}$$

In this formula, μ is a constant and its size influences the length of iterative approaches on the steepest direction each time. It can be proved that through iterative, the optimal solution $X_N{}^*$ can be got from any initiate $X_N(0)$ with an appropriate μ, which is

$$\lim_{n\to\infty} X_N(n) = X_N{}^* \tag{11}$$

When adopting steepest gradient method to iterative calculate and get the smallest optimal solution, it is needed to calculate the estimated value $\hat{R}_{NN}$ of autocorrelation function R_{NN} and the estimated value $\hat{p}_N$ of cross-correlation function P_N firstly which contain complex matrix calculation, so steepest gradient method is seldom used directly.

In order to reduce the computation of each iterative in getting $X_N{}^*$ further, Widrow has put forward an effective and even simplified method in which he used the estimated value of $E\{e(n)A(n)\}$ as its instantaneous value $e(n)A(n)$, and there has

$$\hat{E}\{e(n)A(n)\} = e(n)A(n) \tag{12}$$

Exporting the approximate realization of steepest gradient method:

$$X_N(n+1) = X_N(n) + 2\mu e(n)A(n) \tag{13}$$

This is the famous LMS algorithm (LMS algorithm in short in the following) from Widrow-Hoff. Concluded rolling optimal steps of filter based on the LMS algorithm are as follows:

① according to the specific applied subjects, initiate $X_N(0)\mu$;

② calculate $e(n) = i(n) - X_N{}'(n+1)A(n+1)$;

③ calculate $X_N(n+1) = X_N(n) + 2\mu e(n)A(n)$;

The first rolling optimal calculation has been finished now, and new calculation cycle comes. It is required to go back to the second step and start the new rolling optimal calculation. It is obviously that the computation between LMS algorithm's symbols is small and only needs (2N+1) times of multiplication and 2N times of adder, together with minor storage which is among its biggest advantage.

Therefore, after fk being determined according to formula (13), X becomes a constant coefficient matrix and the amplitude and phase of harmonic wave can be realized easily through software programs. When adopting actual sampling of current into calculation, the best vector A can be obtained under such situation. When A is existing, combination of cosine sequence should be employed rather than combination of complex exponential into practical applications because of the current sampling sequence is real numbers. That is

$$\bar{i}n = \sum_{k=1}^{m} B_k \cos(\omega_k n\Delta T + \theta k)$$

$$n = 0,\ 1 \ldots\ldots N-1 \tag{14}$$

Because $\cos(x) = (e^{jx} + e^{-jx})/2$

X=

$$\begin{bmatrix} e^{j\omega_1 0} & e^{j\omega_2 0} & \cdots & e^{j\omega_m 0} & e^{-j\omega_1 0} & e^{-j\omega_2 0} & \cdots & e^{-j\omega_m 0} \\ e^{j\omega_1 \Delta t} & e^{j\omega_2 \Delta t} & \cdots & e^{j\omega_m \Delta t} & e^{j\omega_1 \Delta t} & e^{j\omega_2 \Delta t} & \cdots & e^{j\omega_m \Delta t} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ e^{j\omega_1 (N-1)\Delta t} & e^{j\omega_2 (N-1)\Delta t} & \cdots & e^{j\omega_m (N-1)\Delta t} & e^{-j\omega_1 (N-1)\Delta t} & e^{-j\omega_2 (N-1)\Delta t} & \cdots & e^{-j\omega_m (N-1)\Delta t} \end{bmatrix}$$

$$A = (a_1 a_2 \ \cdots \ a_m \ \bar{a}_1 \ \bar{a}_2 \ \cdots \ \bar{a}_m)^T$$

There has

$$A = (X^H X)^{-1} X^H I \tag{15}$$

Among it

$$a_k = \frac{1}{2} B_k e^{j\theta_k} \qquad \bar{a}_k = \frac{1}{2} B_k e^{-j\theta_k} \tag{16}$$

Amplitude and phase of every sub-harmonic wave can be obtained according to formula (16). This result plays a crucial role in application of frequency control which used in the design of controllers, and the reason why frequency control is necessary is determined by two factors. First of all, the paralleled active part is either be connected with one or several passive filter slips directly or be paralleled with other passive slips in order to improve filtering effect and do some reactive power compensation at the same time. Therefore, the control to certain number of harmonic waves is unnecessary and it brings waste to compensation capacity and reduces filtering effect of passive slip. Secondly, during the process of injecting harmonic compensation into grid, the amplitude changes and phase shifts of each harmonic wave are differential and needing to be put into consideration through frequency division.

3. Simulation and Analysis of Detective Algorithm

Taking advantages of Matlab software to simulate and analyze the current sub-band detection which is in order to improve adaptive frequency tracking based on extension Prony Spectrum Estimation method, it can be learned:

3.1 Simulation and analysis I

The selected signal is:

$$i(t) = 4\sin(\omega_0 t) + 3\sin(3\omega_0 t + \pi/3),$$

Supposing needing isolation of fundamental and three harmonic waves, then set the sampling frequency of detective system as 400Hz, and the sampling point 64, and the detailed process is as follows:

Firstly, according to formula (3-15) of adaptive frequency tracking algorithm, the frequency voltage is 50HZ at that point and then calculate matrix X:

X=

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.9988-j0.0491 & 0.9892-j0.1467 & 0.9988+j0.0491 & 0.9892+j0.01467 \\ 0.9952-j0.098 & 0.9569-j0.2903 & 0.9952+j0.098 & 0.9569+j0.2903 \\ 0.9892-j0.1467 & 0.904-j0.4276 & 0.9892+j0.1467 & 0.904+j0.4276 \\ 0.9808-j0.1951 & 0.8315-j0.556 & 0.9808+j0.1951 & 0.8315+j0.5556 \\ 0.97-j0.243 & 0.741-j0.6716 & 0.97+j0.243 & 0.741+j0.6416 \end{bmatrix}$$

According to matrix X, it can be got that:

$(X^H X)^{-1} X^H =$

$$\begin{bmatrix} 0.226-j1.455 & 0.256+j2.010 & -0.178+j1.151 & 0.107-j1.159 & 0.240-j2.012 & -0.134+j1.45 \\ -0.221+j0.444 & 0.257-j0.640 & 0.177-j0.355 & -0.107+j0.382 & -0.240+j0.647 & 0.114-j0.477 \\ 0.226+j1.455 & -0.256-j2.010 & -0.178-j1.151 & 0.107+j1.159 & 0.240+j2.012 & -0.114+j0.477 \\ -0.221-j0.444 & 0.257+j0.640 & 0.177+j0.355 & -0.107-j0.382 & -0.240-j0.647 & 0.114+j0.477 \end{bmatrix}$$

Finally, very initiate 6 points among signals can form vector I, therefore there are less sampling points than using FFT and the computational speed has been improved. Thus vector A can be obtained:

$$A=(X^H X)^{-1} X^H I=\begin{bmatrix} j\,2.000 \\ 1.299+j0.75 \\ -j\,2.000 \\ 0.1299-j0.75 \end{bmatrix} \tag{17}$$

It can be determined from formula (17) that the initial phase of fundamental is 90^0 with amplitude 4 while three harmonic waves have an initial phase in 60^0, and 3 in amplitude. Actually, under the situation that having the same frequency resolution and meeting Nyquist's sampling rate, this algorithm needs less sampling points compared with FFT(the sampling points of FFT should meet $N > f_s/(\Delta f)$, and it only has to satisfy the moderate value of N>2P(P is an effective rank). Detailed comparison can be seen in figure 1. By the way, the main calculating process of this algorithm is finished by another DSP processor. Thus, less time and higher accuracy are embodied in the detection.

Table 1. The result of adaptive frequency tracking's current sub-frequency detection based on the extension Prony Spectrum Estimation

Current signal (Prony) the results of the analysis (64 points)		
Current(A)	Frequency(Hz)	Early phase (rad)
4.0000	50.0000	1.5708
4.0000	-50.0000	-1.5708
3.0000	150.0000	-0.5236
3.0000	-150.0000	0.5236

Table 2. The result of current signals based on FFT

Current signal of the FFT analysis (256points)		
Current (A)	Frequency (Hz)	Early phase (rad)
4.0273	50.0000	1.5516
2.9948	150.0000	0.5271

4. Conclusion

This paper presents an adaptive frequency tracking current detection method based on an improved extension of the Prony's for spectrum estimation. The algorithm not only provides a novel method for real-time detection, but also takes into consideration of the slow voltage change in the power grid and the possible mutation of first harmonics of the current. Simulation results demonstrate that this new adaptive detection method meets the real-time and accuracy requirements in detecting the fundamental and first harmonics in a power grid. It reaches a satisfying compromise in achieving both accurate detection and rapid response time.

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