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A COMPREHENSIVE MARKET-BASED SCHEME FOR VAR MANAGEMENT AND PRICING IN THE ELECTRICITY MARKETS

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ABSTRACT

The present paper proposes an integrated market-based approach for pricing VAR service in the electricity market. The structure of the market composed of VAR capacity and VAR utilization during system possible transition states. The possible transition states considered here are multiple base cases and contingencies with their associated occurrence probabilities. The market is based on a long-term contract, where the successful candidates will be remunerated for the provision of VAR capacity as well as VAR utilization during the contracted period. The problem is stated as a minimization problem so that financial and technical issues, emphasizing voltage security issue, are regarded explicitly in a unified single formulation.

KEY WORDS

Deregulation, VAR market, voltage security, PSO.

1. Introduction

The provision of VAR ancillary services from the VAR sources in electricity markets is critical and quite effective to enable power system to operate within an acceptable degree of reliability and security [1]. The procurement of VAR services is especially challenging for transmission operator "TO" in the aspects related to pricing mechanism and several technical issues during system operation. TO should employ a pricing mechanism that enables it to procure VAR services in a minimum payment, with insignificant economical impact on market players. Meanwhile, TO should also recognize the critical VAR providers and fairly remunerate them according to their relative worth for the system security. The technical issues that should be taken into consideration in the procurement of VAR services include the following:

- Possible power system transition states with their associated occurrence probabilities.
- Adequate VAR capacity that should be available for each state to ensure system security.
- Minimization of the VAR utilization during system operation to guarantee low economical effect of this service.

In the existing markets, it has been noted that most of transmission operators address VAR procurement challenges through long-term planning in two pricing approaches. The first approach is cost-based payment such as New York and PJM markets and the second one is market-based pricing such as UK market. The acquiring of VAR support services in these markets mainly relies on the heuristics and TO' judgments and the above technical issues have not considered clearly in their VAR services management. Consequently, adequate security level, fairly remuneration of VAR providers and lowest payment of VAR services can't be guaranteed in these pricing schemes. Recently, several research studies have been presented to tackle the deficiencies of the existing pricing mechanisms based on the long term contracts and a day–head market [2-5]. In spite of their significant contributions, a pricing proposal that considers the above financial and technical issues in unified single problem has not been yet developed, which is the concern of this paper.

The present paper is an extension of the authors' proposal for the provision of the VAR service from dynamic VAR sources in a competitive market-based environment [6]. The formulation has been modified to include VAR utilization payment and possible power system transition states "multiple base cases and contingencies" with their associated occurrence probabilities. This treatment permits to accommodate real power system circumstances and consequently evaluate realistically expected total VAR capacity and utilization payment during the contracted period. The problem is stated as a minimization problem so that financial and technical issues mentioned above, emphasizing voltage security issue, are regarded explicitly in a unified single formulation. The objective function, which is the sum of expected VAR capacity payment, VAR utilization payment and operating costs during system operation, is assessed probabilistically under possible power system transition sates. The proposed method is suited for the existing UK VAR market, where it can be employed for the simulation and analysis of such kind of VAR market arrangements.

2. Basic Terms of the Proposed Approach

2.1 VAR Market Objective

In this section, the basic concept of the proposed VAR procurement method is presented. Fig.1 is assumed to illustrate the intrinsic idea behind this work. First, we suppose that TO invites VAR providers to participate in its VAR market, where the main providers are generators and synchronous condensers. The structure of this market composed of VAR capacity and VAR utilization during system transition states. Then, the main target of TO is to get long term contracts with most beneficial VAR providers. The most beneficial providers are those that simultaneously ensure system security during expected operating sates and minimize expected total TO VAR service payment. Achieving this target requires TO to specify a set of expected operating conditions with their possibilities during the contracted period. Based on the power system transition states discussed above, a set of possible operating conditions that TO may employ for this market is assumed as given in Fig. 1. It is assumed that, during contracted period, there are a number of load levels "$L^{(1)}$, $L^{(2)}$, …, $L^{(T)}$" that TO considers significant for the analysis and simulation in this market. The corresponding time durations of these load levels are "$D^{(1)}$, $D^{(2)}$, …, $D^{(T)}$", while the associated base cases are "$A^{(1)}$, $A^{(2)}$, …, $A^{(T)}$" as indicated in Fig.1. It is also supposed that, for each load level, there are a number of contingencies N the system may be exposed for. At the load level $L^{(t)}$, when a contingency k occurs with probability $\alpha^{(k,t)}$, the system will proceed to the corrective state $B^{(t)}$. Therefore, the probability that the system will be in base case operating state at load level $L^{(t)}$ is $(1-\sum\alpha^{(t,k)})$. According to this assumption, for load level $L^{(t)}$, the number of hours that will be spent in corrective state corresponding to contingency k is $D^{(t)}\ \alpha^{(k,t)}$, while the number of hours that will be spent in base case is $D^{(t)}\ (1-\sum\alpha^{(t,k)})$. The problem now is how to procure a minimum VAR capacity that accommodates all of these operating sates and maintain a certain degree of security for each individual state. Another problem is how to ideally utilize this VAR capacity during operation of each expected state and consequently minimize whole VAR service payment. For this purpose, the objective function is adopted to simultaneously minimize VAR capacity payment, expected VAR utilization payment and operating costs under all transition sates as described by the following equation

$$F_{Total} = F_{Cap} + \sum_{t=1}^{T} F^{(t)}, \quad F^{(t)} = D^{(t)}(F_A^{(t)} + F_B^{(t)}) \qquad (1)$$

$$F_A^{(t)} = (1 - \sum_{k=1}^{N} \alpha^{(k,t)})(F_{UA}^{(t)} + F_{Opt}^{(t)})$$

$$F_B^{(t)} = \sum_{k=1}^{N} \alpha^{(k,t)}(F_{UB}^{(k,t)} + F_{Bc}^{(k,t)})$$

where F_{Total} is the total objective function, F_{Cap} is the VAR capacity payment; $F^{(t)}$ is expected operating cost of the load level $L^{(t)}$; $F_A^{(t)}$ and $F_B^{(t)}$ are the expected operating costs of the base case and corrective states for the load level $L^{(t)}$; $F_{UA}^{(t)}$ and $F_{Opt}^{(t)}$ are the base case VAR utilization payment and power loss cost for load level $L^{(t)}$; $F_{UB}^{(k,t)}$ and $F_{Bc}^{(k,t)}$ are the VAR utilization payment and corrective control costs for load level $L^{(t)}$ and contingency k. A detailed description of each individual objective function and its associated constraints that have been employed to ensure system security for all the above operating states will be explained hereafter.

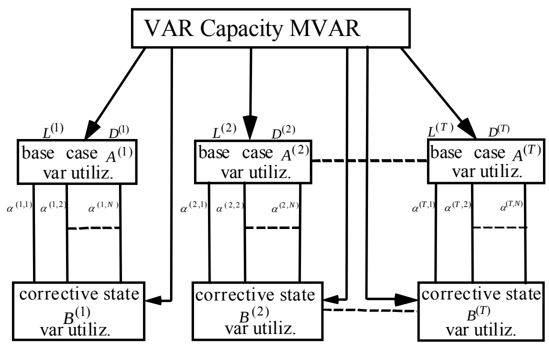

Fig. 1: Basic concept of the proposed VAR market

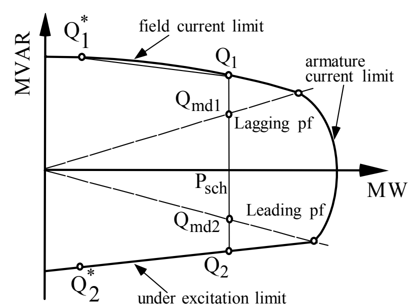

Fig. 2: Generator Capability Curve

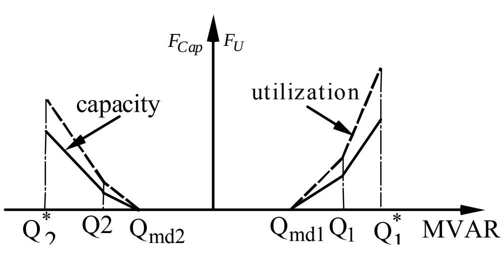

Fig. 3: Payment Structure

2.2 Generator VAR Output and its Compensation

The VAR market scheme presented here considers only the generators and synchronous condensers and depends mainly on the generator capability curve shown in Fig. 2. In this paper we assume that each generator will provide its VAR service as described in following regions:

Region I (Q_{md2} *to* Q_{md1})*:* The reactive power produced in this region is obligatory with no payment.

Region II (Q_{md1} *to* Q_1 *&* Q_{md2} *to* Q_2)*:* This region represents the extra reactive VAR provided by generator beyond its obligatory without rescheduling its real power output. A generator in this region is expecting a payment from the TO for its service.

Region III (Q_1 *to* Q_1^* *&* Q_2 *to* Q_2^*)*:* In this region the generator will reduce its real power schedule (P_{sch}) and consequently its lost revenue will be recovered by the TO. This payment is known as opportunity cost payment. The adjustment of the real power schedule corresponding to VAR output can be computed based on the slope of line segment $Q_1\ Q_1^*$ or $Q_2\ Q_2^*$ since the data of Fig.2 assumed to be submitted to TO in the proposed scheme.

Based on the classification of the above regions, a bidding scheme that allows the TO to procure VAR service from generators and synchronous condensers in competitive manner is introduced. This market consists of VAR capacity and VAR utilization during the expected transition states. Therefore, the VAR providers will provide their VAR capabilities in MVAR and their associated offer prices in \$/MVAR for the recovery of the VAR capacity. Also they are required to submit their VAR utilization offer prices in \$/MVARh in order to recover the VAR utilization during system operation. The bidding method mainly relies on the generator VAR payment function depicted in Fig. 3.

The mathematical expression of the VAR capacity payment F_{Cap} is given by equation (A1) in Appendix A. The VAR capacity pattern acquired based on (A1) will be utilized in the normal state and emergency situations, where each successful provider will make its contracted VAR capacity available for the TO to be employed during system operation. For the recovery of the VAR utilization, the bidding criterion is identical to the VAR capacity payment. Namely, the generators will provide their offer prices in \$/MVARh for each region discussed above and the utilization payment will be determined according to the VAR utilized in the system operation based on the VAR utilization payment equation given in Appendix B.

3. Transition States Sub-problems

In this section, the mathematical formulation that considers VAR capacity payment, utilization payment and operating costs under the previous transition states in a unified single problem is introduced. The multi-transition states that have been introduced in our previous work for the conventional VAR planning problem [7-8] are exploited here.

3.1 Base Case Sub-problems

The base case sub-problems evaluate the operation cost of the normal stats under specific number of load levels stipulated by TO. For each load level, the power system is supposed to operate for a certain period of time. Therefore, choosing a proper objective function to be minimized in the normal operation throughout duration time of each load level can effectively satisfy adequate payment of VAR service. In this paper, the cost of the power loss and VAR utilization payment are selected as the main objective function in each base case sub-problem. To maintain voltage stability margin, two sets of constraints have been included in the formulation for each base case. The first set represents the equality and inequality constraints at the nominal load operating point and the second set represents the equality and inequality constraints at the point of collapse. According to this assumption, the base case sub-problem of the load level $L^{(t)}$ is formulated as:

$$F_A^{(t)} = (1 - \sum_{k=1}^{N} \alpha^{(k,t)})(F_{UA}^{(t)}(Q_b^{(0)}) + F_{Opt}^{(t)}(x_b^{(0)}, p_b^{(0)}, Q_b^{(0)})) \quad (2)$$

subject to

$$\left.\begin{array}{l} y_b - (r_2 + r_4)dp_{sch} - s_b^{(0)} - f\,(x_b^{(0)}, p_b^{(0)}, Q_b^{(0)}) = 0 \\ 0 \le s_b^{(0)} \le s_{\max}\ , \qquad x_{\min} \le x_b^{(0)} \le x_{\max} \\ p_{\min} \le p_b^{(0)} \le p_{\max}\ , \quad Q_{\min} \le Q_b^{(0)} \le Q_{\max} \end{array}\right\} \quad (3)$$

$$\left.\begin{array}{l} y_b + (\lambda_c^{(0)} - 1)y_d - (r_2 + r_4)dp_{sch} - s_c^{(0)} - f\,(x_c^{(0)}, p_c^{(0)}, Q_c^{(0)}) = 0 \\ w\,(x_c^{(0)}, p_c^{(0)}, Q_c^{(0)}, s_c^{(0)}, \lambda_c^{(0)}) f_x\,(x_c^{(0)}, p_c^{(0)}, Q_c^{(0)}, s_c^{(0)}, \lambda_c^{(0)}) = 0 \\ \|w\| \ne 0 \\ 0 \le s_c^{(0)} \le s_{\max}\ , \qquad \lambda_c^{(0)} \le \lambda_{\min} \\ p_{\min} \le p_c^{(0)} \le p_{\max}\ , \quad Q_{\min} \le Q_c^{(0)} \le Q_{\max} \end{array}\right\} \quad (4)$$

where

$$Q_{\min} = Q_{md2} + r_1 Q_{g1} + r_2(Q_2 - Q_{md2}) + r_2 Q_{g2}$$
$$Q_{\max} = Q_{md1} + r_3 Q_{g3} + r_4(Q_1 - Q_{md1}) + r_4 Q_{g4}$$

where (3) with the subscripts b and (4) with c indicate the nominal load operating point and collapse point respectively. Constraints (3) consist of ac power flow equations, operation limits of voltage magnitude, angle, load shedding, VAR devices including generators, synchronous condensers, etc. Constraints (4) consist of conditions for voltage collapse point, which include a set of point of collapse equations, limits of control devices, load shedding and load power margin for security. $F_{opt}^{(t)}$ is power loss cost for load level $L^{(t)}$. $F_{UA}^{(t)}$ is base case VAR utilization payment for load level $L^{(t)}$. The superscript (0) refers to the base case sub-problem. x is the state variables vector " voltage magnitudes and angles". s is load shedding vector. p is the control variables vector

"VAR control devices" excluding Q . y_b is nominal load " base case". y_d is load direction vector. Q is the generator and synchronous condensers VAR output. f is power flow equations at nominal load. w is left eigenvector " row vector". f_x is power flow Jacobian " singular at nose point". dp_{sch} is the change of active power schedule in region III. λ is the load parameter value.

Note that hat the term $(r_2+r_4)dp_{sch}$ in (3) and (4) will only be active when generator provides its VAR in region III. Otherwise this term will be null.

In the above formulation, in order to simplify the problem, the utilization payment $F_{UA}^{(t)}$ is expressed as a linear function of the reactive power output of each provider "$Q_b^{(0)}$". This simplification enables us to treat each base case sub-problem in the implementation as nonlinear programming problem as we will discuss hereafters. Consequently, based on the output of the reactive power $Q_b^{(0)}$, the var utilization and its associated payment is calculated. For instance, when the lagging var output $Q_b^{(0)}$ is between zero and Q_{md1}, the reactive power utilization Q_{u3} and Q_{u4} are equal zero and consequently $F_{UA}^{(t)}$ is equal zero. When the lagging VAR output $Q_b^{(0)}$ is between Q_{md1} and Q_1, the VAR utilization Q_{u4} is equal zero and the reactive utilization Q_{u3} and it associated payment are computed based on the linear segment $Q_{md1}\,Q_1$. Finally, when the lagging VAR output $Q_b^{(0)}$ is between Q_1 and Q_1^*, $Q_{u3} = Q_1 - Q_{md1}$, while Q_{u4} and its associated payment are calculated based on the linear segment $Q_1\,Q_1^*$.

3.2 Post-Contingency States Sub-problems

As indicted in Fig.1, for each load level $L^{(t)}$, there are a number of contingencies N that proceed the system to the corrective states. Each contingency will be remained in the corrective state for a certain period of time. The main objective here is to employ a proper objective function that ensures a minimum VAR services payment in these sates while maintaining system security. To achieve this purpose, for each contingency state, the corrective control actions are assumed based on the reactive power controls and load shedding to guarantee the system security. We assume that the VAR control costs are trivial compared with the load shedding cost. The objective function is chosen to minimize simultaneously the expected total amount of the control costs and VAR utilization payment while satisfying the constraints set for the nominal load operating point and the collapse point. The formulation of this problem for the load level $L^{(t)}$ is stated as:

$$\text{Minimize } F_B^{(t)} = \sum_{k=1}^{N} F_B^{(k,t)} \tag{5}$$

$$F_B^{(k,t)} = \alpha^{(k,t)}\left(F_{UB}^{(k,t)}(Q_b^{(k)}) + F_{Bc}^{(k,t)}(p^{(0)}, p^{(k)}, s^{(k)}, Q^{(0)}, Q^{(k)})\right),$$

$$F_{Bc}^{(k,t)} = \left\{\sum_l \mu_{sl}\left|s^{(k,t)}\right| + \sum_i \mu_{pi}\left|p^{(k,t)} - p^{(0)}\right| + \sum_j \mu_{qj}\left|Q^{(k,t)} - Q^{(0)}\right|\right\}$$

subject to

$$\begin{aligned} &G_b^{(k,t)}(x_b^{(k,t)}, p_b^{(k,t)}, s_b^{(k,t)}, Q_b^{(k,t)}, \lambda_b^{(k,t)}) \le 0 \\ &G_c^{(k,t)}(x_c^{(k,t)}, p_c^{(k,t)}, s_c^{(k,t)}, Q_c^{(k,t)}, \lambda_c^{(k,t)}) \le 0 \end{aligned} \quad k=1:N \tag{6}$$

where $G_b^{(k,t)}$ and $G_c^{(k,t)}$ are similar to the constraints (3) and (4) respectively except that the superscript k refers to post- contingency state and the load shedding s is included. μ_{sl}, μ_{pi} and μ_{qj} are unit control cost coefficients of s, p and Q respectively. $F_{UB}^{(k,t)}$ and $F_{Bc}^{(k,t)}$ are VAR utilization payment and corrective control cost for the load level $L^{(t)}$ and contingency k. Similar to the base case sub-problems, based on the output of the reactive power $Q_b^{(k,t)}$, the VAR utilization Q_{u1}, Q_{u2}, Q_{u3} and Q_{u4} and its associated payment $F_{UB}^{(k,t)}$ will be determined.

4. Overall Problem Formulation

To guarantee the economic efficiency of the VAR service, we simultaneously minimize the total payment of procured VAR and operating costs in all transition states as follows:

$$\text{Minimize } F_{Total} = F_{Cap} + \sum_{t=1}^{T} D^{(t)}(F_A^{(t)} + F_B^{(t)}) \tag{7}$$

$$\text{Subject to} \begin{cases} \text{Generator constraints (A2- A4)} \\ \text{Base case constraints (3) and (4)} \\ \text{Post-contingency states constraints (6)} \end{cases}$$

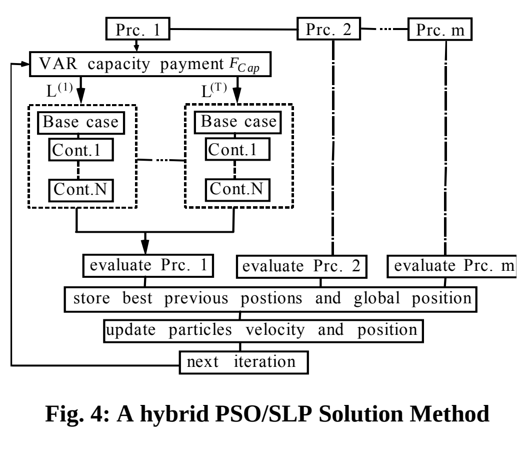

Fig. 4: A hybrid PSO/SLP Solution Method

5. Solution Algorithm

The overall problem (7) is deemed as a large-scale mixed integer nonlinear optimization problem. An optimization technique based on a hybrid particle swarm optimization/successive linear programming (PSO/SLP) for finding a global optimal solution of (7) is presented in this section. The computational procedures of the proposed method are summarized in Fig.5. The algorithm starts from a random initial swarm, where its particles are indicated in Fig. 5 by Prc.1, Prc.2.,…., Prc.1,. Each particle in the swarm represents a candidate solution, i.e., a pattern of generators VAR capacity. For instant, assume particle 1 (Prc.1) represents a candidate pattern of generators VAR capacity, where its payment F_{Cap} can directly computed using equation (A1). This candidate pattern is used as a common candidate for each transition state in the load levels "$L^{(1)}$, $L^{(2)}$, …, $L^{(T)}$" to minimize operating costs and VAR utilization payment during normal operation and emergency states. For each load level, the SLP is used to solve individually the base case optimization sub-problem (2-4) and its associated post-contingency states sub-problems (5-6). The expected operating costs "$F^{(1)}$, $F^{(2)}$, …,$F^{(T)}$" of the load levels "$L^{(1)}$, $L^{(2)}$, …,$L^{(T)}$" are computed in these optimization problems. According to the optimization results, the fitness of prc.1 is evaluated in terms of F_{Cap} , $F^{(1)}$, $F^{(2)}$, …and $F^{(T)}$. The same computational procedures will be repeated for each particle in the swarm. Consequently, the best previous position for each particle and best particle among all the particles are stored in a solution set. Then, the new velocity and position for each particle are updated based on current velocity, current position, producing next iteration. These procedures are repeated till a termination criterion is satisfied.

6. Simulation Results

The proposed method of VAR market scheme was tested on modified IEEE 57 bus system. The analysis were executed for three load levels ($NL=3$) at 130%, 140 % and 150 % of the original load. The corresponding time durations (T) of the three load levels are set 70%, 20% and 10% respectively. Two severe contingencies have been adopted for each load level for the examination. The severe contingencies of 130% load level were the outages of lines (25-30) and (46-47) with probabilities 0.03 and 0.025 respectively. The outages of lines (25-30) and (46-47) with probabilities 0.02 and 0.015 are assumed for the load level 140%. The associated contingencies and probabilities for the third load level 150% are the outages of lines (25-30) and (46-47) with probabilities 0.01 and 0.005 respectively.

The objective of TO in this simulation is to get a long term contract with the promising VAR providers in a minimum payment while keeping the load margin ≥ 0.25 and bus voltage magnitudes within 0.9 pu and 1.1 pu. The minimum payment means that a simultaneous minimization of the expected VAR capacity and VAR utilization payment under the previous transition states. The period of long tem contract is assumed 180 days. According to the contracted period and the data given above the time duration of each transition state is indicated in table 1.

Table 1: Time durations of the transition states "hours"

Load Level	1.3	1.4	1.5
Base case	2786.4	712.8	367.2
Cont 1	129.6	86.4	43.2
Cont 2	108.0	64.8	21.6

Table 2 shows the offered prices in \$/MVAR and \$/MVARh for the recovery of the VAR capacity as well as VAR utilization during system operation respectively. The VAR capabilities of each region associated with each load level are also indicted in the table. The data given in table 1 is provided for only the lagging region which is vital for the voltage stability problem. Note that the providers 1, 3 and 5 are synchronous condensers and therefore their VAR mandatory obligations and opportunity offer prices are set zero as shown in Table 2.

Table 2: Generators and synchronous condensers offers

		Generator	1	2	3	4	5	6
Capacity prices		μ_3 , μ_4	24,0.0	21, 37.5	18,0.0	25.5,34.5	17.4,0.0	23.4,42
Utilization prices		μ_{u3} , μ_{u4}	0.016, 0.0	0.014, 0.05	0.012, 0.0	0.017, 0.046	0.012, 0.0	0.016, 0.056
Load level	1.3	Q_{md1} , Q_1 , Q_1^*	0.0, 0.5, 0.5	0.17, 0.79, 0.97	0.0, 0.25, 0.25	1.92, 2.77, 3.25	0.0, 0.09, 0.09	1.32, 2.07, 2.52
	1.4	Q_{md1} , Q_1 , Q_1^*	0.0, 0.5, 0.5	0.18, 0.77, 0.97	0.0, 0.25, 0.25	2.07, 2.63, 3.25	0.0, 0.09, 0.09	1.42, 2.00, 2.52
	1.5	Q_{md1} , Q_1 , Q_1^*	0.0, 0.5, 0.5	0.20, 0.75, 0.97	0.0, 0.25, 0.25	2.22, 2.50, 3.25	0.0, 0.09, 0.09	1.53, 1.94, 2.52

Based on the data submitted in table 2, the solution algorithm given in section 4 is executed. The parameters of PSO used in the simulation are: $\omega_{\min}$ =0.4, $\omega_{\max}$ = 0.9, $c_1 = c_2$ =2, $v_{id\ \min}$ =-2 $v_{id\ \max}$ = 2, swarm sizes 20.

The optimal VAR procured from the VAR providers 1 to 6 are 0.193, 0.789, 0.25, 2.51 and 1.94 respectively. The associated total cost is 233.75. The convergence characteristic for this examination is given in Fig.5.

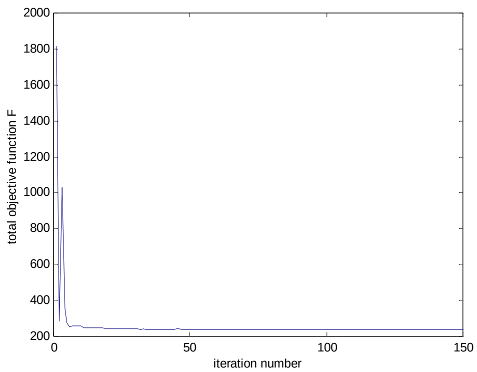

Fig. 5: convergence characteristic of PSO/SLP

Table 3: VAR utilization payments and operating costs

	Load Level	1.3	1.4	1.5
VAR Utilization Payment	Base case	49.6546	15.9696	9.6945
	Cont 1	2.3000	2.0177	1.1035
	Cont 2	1.8779	1.5138	0.5643
Operating Costs	Base case	66.1655	20.0624	12.2990
	Cont 1	0.0000	0.0000	0.0000
	Cont 2	0.0000	0.0000	0.0000

The procured VAR are used to maintain the desired minimum voltage magnitude "0.9" and load margin value "0.25" during operation for all the expected transition states "base cases and contingency states". The total cost stands for the VAR capacity payment, VAR utilization payment and the operating costs "power losses and control costs". The total capacity payment is 50.52, which represents the sum of capacity payments associated with above load levels and their expected contingencies. The capacity payment for the load levels 1.3, 1.4 and 1.5 are 37.08, 9.36 and 4.08 respectively. Note that, the more the load level is, the lesser its capacity payment. That is occurred as a results of the increasing of real power schedules for the generators and consequently their VAR mandatory obligations are increased to ensure the transfer of the real power. According to the expected time duration given in table 1, the procured VARs are exploited during system operation for all the transition states. The VAR utilization payments and operating costs corresponding to each state are shown in table 3. Observe that, the load level 1.3 has the highest base case utilization payment since its time duration is much higher than the load levels 1.4 and 1.5. Note also that, since the time durations for the contingency cases are much lower than the base cases, the utilization payments are too low compared to the base cases for all load levels. The operating costs are mainly the base case costs which stand for the power losses costs associated with each load level. The control costs are almost zero since their unit costs are set too low in the simulation.

7. Conclusion

An integrated scheme which considers both of VAR capacity and VAR utilization payments is introduced. The financial and technical issues, emphasizing voltage security issue, are regarded explicitly in a new unified single formulation. The objective function, which is the sum of expected VAR capacity payment, VAR utilization payment and operating costs during system operation, is assessed probabilistically under possible power system transition sates "multiple base cases and contingencies". The method has been tested on IEEE-57 bus system [7], where the results demonstrate its rigorous applicability. The proposed method is suited for the simulation and analysis of existing UK VAR market.

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Appendix A: VAR Capacity Payment Formulation

The mathematical expression of the VAR capacity payment is given by the following equation:

$$\begin{aligned} F_{Cap} &= (-\mu_1 Q_{g1})r_1 - \mu_1(Q_2 - Q_{md2})r_2 - (\mu_2 Q_{g2})r_2 \\ &+ (\mu_3 Q_{g3})r_3 + \mu_3(Q_1 - Q_{md1})r_4 + (\mu_4 Q_{g4})r_4 \end{aligned} \quad \text{(A1)}$$

With the constraints A2 and A3 representing leading and lagging regions respectively.

$$r_1(Q_2 - Q_{md2}) \le Q_{g1} \le 0,\ r_2(Q_2^* - Q_2) \le Q_{g2} \le 0 \quad \text{(A2)}$$

$$0 \le Q_{g3} \le (Q_1 - Q_{md1})r_3,\ 0 \le Q_{g4} \le (Q_1^* - Q_1)r_4 \quad \text{(A3)}$$

$$r_1 + r_2 + r_3 + r_4 \le 1 \quad \text{(A4)}$$

where the coefficients (μ_1, μ_3) and (μ_2 , μ_4) are the offer prices in \$/MVAR that the generators provide for regions II and III respectively; (Q_{g1}, Q_{g3}) and (Q_{g2}, Q_{g4}) are variables to be determined corresponding to provided VAR amounts in regions II and III respectively; Q_1, Q_1^*, Q_2 and Q_2^* are parameters to be offered by the generators; r_1, r_2, r_3 and r_4 are binary variables. According to (A4) only one of these binary variables can be selected. This constraint ensures that VAR output of generators will be in only one of the defined three regions.

Appendix B: Formulation of VAR Utilization Payment

The VAR utilization payment is represented mathematically as follows:

$$\begin{aligned} F_U &= (-\mu_{u1} Q_{u1})r_1 - \mu_{u1}(Q_{u1})r_2 - (\mu_{u2} Q_{u2})r_2 \\ &+ (\mu_{u3} Q_{u3})r_3 + \mu_{u3}(Q_{u3})r_4 + (\mu_{u4} Q_{u4})r_4 \end{aligned} \quad \text{(B1)}$$

With the constraints B2 and B3 representing leading and lagging regions respectively.

$$r_1 Q_{g1} \le Q_{u1} \le 0,\ r_2 Q_{g1} \le Q_{u1} \le 0,\ r_2 Q_{g2} \le Q_{u2} \le 0 \quad \text{(B2)}$$

$$0 \le Q_{u3} \le r_3 Q_{g3},\ 0 \le Q_{u3} \le r_4 Q_{g3},\ 0 \le Q_{u4} \le r_4 Q_{g4} \quad \text{(B3)}$$

where the coefficients (μ_{u1}, μ_{u3}) and (μ_{u2} , μ_{u4}) are the offer prices in \$/MVARh that the generators provide for regions II and III respectively; (Q_{u1}, Q_{u3}) and (Q_{u2}, Q_{u4}) are the utilized VAR amounts to be determined in regions II and III respectively. In the above equations, the constraints (B2) and (B3) guarantee the VAR utilization variables Q_{u1}, Q_{u2}, Q_{u3} and Q_{u4} to be within the committed VAR capacity Q_{g1}, Q_{g2}, Q_{g3} and Q_{g4} for each individual region respectively.