International Journal of Robotics and Automation, Vol. 36, 2021

SLIDING MODE VARIABLE STRUCTURE OBSERVER-BASED SENSOR AND ACTUATOR FAULT RECONSTRUCTION FOR NONLINEAR SYSTEM

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Abstract

This paper presents the design of a sliding mode variable structure observer, which is used for the fault detection and isolation of sensors and actuators in uncertain nonlinear systems. Firstly, the coordinate of a system output equation is transformed, and a sensor fault is equivalent to an actuator one by using a low-pass filter. Then using a coordinate transformation to transform the equivalent equation into three sub-systems, and the uncertainties only exist in the second one, while the first contains actuator faults and the third has equivalent actuator faults. The design of three observers, respectively, corresponding to the above sub-systems, the actuator faults, sensor faults as well as the unknown input disturbances are reconstructed separately as a result. Two simulation examples confirm the truth of effectiveness of the reconstruction plan.

Key Words

Fault reconstruction, sliding mode variable structure, observers

1. Introduction

System faults and unknown input disturbances are inevitable during operation of a complicated power system. With regard to physical position, faults mainly include actuator fault and sensor fault. Faults can damage the normal system operation and lead the system to an unstable state. Therefore, the fault detection and isolation (FDI) technique is significant in system operation. In the past decades, the study on FDI made great progress, especially for the model-based fault detection [1]–[3]. Various approaches have been presented to solve FDI problems, such as differential geometry method, self-adaptive control method, and sliding mode variable structure observer technique [4], [5]. Moreover, research on fault tolerance control and stability analysis considerably promoted the FDI [6]–[8]. The power system used in this study inevitably suffers from unknown input disturbances and potential system faults during operation. The sliding mode control technique, as an effective method with high robustness and insensitivity to system uncertainty, offers great potential for robust FDI. Therefore, it has been extensively employed in the study for FDI [9]–[11].

Faults reconstruction method that employs sliding mode techniques has become an important field, which mainly includes actuator fault reconstruction, sensor fault reconstruction, and fault reconstruction considering unknown input disturbances [12]–[15]. Different from an actuator, a sensor is a passive element that merely provides the information of an operating system and does not directly affect system behaviour. Furthermore, the sensor is located at a feedback channel and thus does not adjust the parameter perturbation and interference through a feedback mechanism like other elements. Minor sensor faults may result in system maloperation and damage the system stability. Thus, study on sensor fault diagnosis is vital to performance improvement of a complete system. At present, studies on sensor fault mainly focus on the linear system [16]. Brahim *et al.* discussed a type of linear system, where a primary transformation was first made on the output equation to obtain two output sub-systems. The first sub-system did not include sensor faults, whereas the second sub-system included the sensor faults. A first-order low-pass filter for the second sub-system was then designed [17]. The study on sliding mode variable structure observer-based nonlinear fault diagnosis mainly focused on actuator fault. There are few study methods on sensor fault diagnosis. Tan and Edwards designed a low-pass filter that transform sensor faults to equivalent actuator ones [18]. Consequently, the study methods on actuator fault diagnosis were used in sensor fault diagnosis. Yan and Edwards proposed a sliding mode observer-based sensor fault reconstruction method by utilizing this transformation [19],

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Recommended by Dr. Daqi Zhu
(DOI: 10.2316/J.2021.206-0629)

and considered two cases. One case was the reconstruction of sensor faults without unknown input disturbance, and the other was the estimation of sensor fault with unknown input disturbance. Alwi *et al.* advanced a new sensor fault reconstruction method for an unstable linear system by designing a sliding mode observer. The concerning sensor faults were slow faults. The system state and sensor faults were integrated as new state variable. The state equation of the transformed system comprised faults of an equivalent actuator as the derivative of sensor faults [20]. Raoufi, Lee *et al.* discussed a type of nonlinear system with actuator faults and sensor faults using a method in the literature [20] and reconstructed and estimated such faults [21], [22].

This paper presents the design of a sliding mode variable structure observer, which is used for the FDI in uncertain nonlinear systems. The basic idea was presented in [23], [24]. Unlike other reported methods, our paper not only considers actuator fault reconstruction, but also considers sensor fault reconstruction. The system output equation was first transformed into two output sub-system. Only the second output sub-system comprised sensor faults. A low-pass filter was designed to transform sensor faults into equivalent actuator faults for the second output sub-system. Linear transformation was then carried out on the system to obtain two sub-systems. After a series of linear transformations and filter design, an original system was transformed to three sub-systems. Sliding mode variable structure observer is designed for each sub-system. The problem of fault identification under disturbances is solved.

In the rest of this paper, Section 2 describes the nonlinear system and presents equation transformation method. Section 3 investigates sliding variable structure observers for the system. Section 4 presents the method for sensor and actuator fault identification. Simulation of two examples is illustrated in Section 5. A conclusion is briefly given in Section 6.

2. Problem Description

Consider a class of systems described by a nonlinear dynamic model:

$$\begin{cases} \dot{x}(t) = Ax(t) + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y(t) = Cx(t) + F_s f_s(t) \end{cases} \tag{1}$$

where $x \in R^n$, $y \in R^p$, and $u \in R^r$ denote the state, outputs, and inputs vectors, respectively. Assume that, $\Phi(x,u) \in R^n$ is a known function vector. $f_a(t) \in R^q f_s(t) \in R^m$ and $d(t) \in R^q$ are unknown function. $d(t)$ signifies the total uncertainty and disturbance of the system. Assuming it is limitary that $\|d(t)\| \le \gamma_1$, where γ_1 is a positive constant. $f_a(t)$ and $f_s(t)$ indicate, respectively, actuator faults and sensor faults, which are also bounded, *i.e.*, two constant γ_2 and γ_3 exist such that $\|f_a(t)\| \le \gamma_2$ and $\|f_s(t)\| \le \gamma_3$. $A \in R^{n\times n}$,$B \in R^{n\times r}$, $C \in R^{p\times n}$, $D \in R^{n\times q}$, $E \in R^{n\times q}$, and $F_s \in R^{p\times m}$ are known constant matrices with $n > p > q + m$.

For the output signal $y \in R^p$ of system (1), a transformation matrix $S_0 \in R^{p\times p}$ exists such that a following equation could be obtained:

$$S_0 \begin{bmatrix} C & F_s \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ C_2 & F \end{bmatrix} \tag{2}$$

we obtain

$$S_0 y = \begin{cases} y_1 = C_1 x \\ y_2 = C_2 x + F f_s \end{cases} \tag{3}$$

where $y_1 \in R^{p-m}$, $y_2 \in R^m$, $C_1 \in R^{(p-m)\times n}$, $C_2 \in R^{m\times n}$, and $F \in R^{m\times m}$ are nonsingular constant matrices. The purpose is to partition the output, so that only y_2 contains potentially sensor fault.

A new state variable $z_a \in R^m$ is then designed as the first-order low-pass filter of the output signal y_2 [17].

$$\dot{z}_a = A_s z_a + B_s y_2 \tag{4}$$

where filter matrices $A_s \in R^{m\times m}$, and $B_s \in R^{m\times m}$ are filter matrices to be designed.

The following equation is obtained by transforming the output signal y_2 of (3) and (4):

$$\dot{z}_a = A_s z_a + B_s C_2 x + B_s F f_s \tag{5}$$

Equation (5) and System (1) are then combined into a new system.

$$\begin{cases} \begin{bmatrix} \dot{x} \\ \dot{z}_a \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_s C_2 & A_s \end{bmatrix} \begin{bmatrix} x \\ z_a \end{bmatrix} + \begin{bmatrix} \Phi(x,u) \\ 0 \end{bmatrix} + \begin{bmatrix} E \\ 0 \end{bmatrix} f_a(t) \\ \qquad + \begin{bmatrix} 0 \\ B_s F \end{bmatrix} f_s(t) + \begin{bmatrix} D \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ \begin{bmatrix} y_1 \\ z_a \end{bmatrix} = \begin{bmatrix} C_1 & 0 \\ 0 & I_m \end{bmatrix} \begin{bmatrix} x \\ z_a \end{bmatrix} \end{cases} \tag{6}$$

A set of new state variables and the corresponding matrix are then designed as follows:

$$\bar{x} = \begin{bmatrix} x \\ z_a \end{bmatrix}, \bar{y} = \begin{bmatrix} y_1 \\ z_a \end{bmatrix} = \begin{bmatrix} y_1 \\ y_a \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ B_s C_2 & A_s \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C_1 & 0 \\ 0 & I_m \end{bmatrix}$$

Giving the following two sub-systems transformed from (6):

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y_1(t) = C_1 x(t) \end{cases} \tag{7}$$

$$\begin{cases} \dot{z}_a = A_s z_a + B_s C_2 x + B_s F f_s \\ y_a = z_a \end{cases} \tag{8}$$

Assumption 1. $rank(CD)=rank(D)$.

Assumption 2. *The* (A,C) *is observable*

Divide (7) into the following form:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \Phi_1(x,u) \\ \Phi_2(x,u) \end{bmatrix} \tag{9}$$

$$+ \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} f(t) + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} d(t) + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t)$$

where $x_1(t) \in R^{n-q}$, $x_2(t) \in R^q$, $\Phi_1(x,u) \in R^{n-q}$, $\Phi_2(x,u) \in R^q$, $E_1 \in R^{(n-q)\times q}$, $E_2 \in R^{q\times q}$, $D_1 \in R^{(n-q)\times q}$, and $D_2 \in R^{q\times q}$ are nonsingular matrices.

Under the Assumptions 1, there exists a transformation of coordinates:

$$x(t) = T^{-1}z(t) = T^{-1}\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} \quad y_1(t) = S^{-1}\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

Therefore, (9) is described by the following equations:

$$\begin{cases} \dot{z}(t) = \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = TAT^{-1}z(t) + T\Phi(z,u) + TEf_a(t) \\ \qquad + TDd(t) + TBu(t) \\ v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = SC_1T^{-1}z(t) \end{cases} \tag{10}$$

where $SCT^{-1} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}$ and C_{22} are nonsingular matrices.

T is a nonsingular matrix satisfying the following structure [23]:

$$T = \begin{bmatrix} I_{n-q} & -D_1D_2^{-1} \\ 0 & I_q \end{bmatrix}$$

Accordingly, the coefficient matrix of (10) has a following form:

$$TAT^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix},\ TD = \begin{bmatrix} 0 \\ \bar{D}_2 \end{bmatrix},\ TB = \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \end{bmatrix}$$

$$TE = \begin{bmatrix} \bar{E}_1 \\ \bar{E}_2 \end{bmatrix},\ T\Phi(x,u) = \begin{bmatrix} \bar{\Phi}_1(x,u) \\ \bar{\Phi}_2(x,u) \end{bmatrix}$$

where $\bar{A}_{11} = A_{11} - D_1D_2^{-1}A_{12}$, $\bar{A}_{12} = (A_{11} - D_1D_2^{-1}A_{21})D_1D_2^{-1} + A_{12} - D_1D_2^{-1}A_{22}$, $\bar{A}_{21} = A_{21}$, $\bar{A}_{22} = A_{21}D_1D_2^{-1} + A_{22}$, $\bar{\Phi}_1(x,u) = \Phi_1(x,u) - D_1D_2^{-1}\Phi_2(x,u)$, $\bar{\Phi}_2(x,u) = \Phi_2(x,u)$, $\bar{E}_1 = E_1 - D_1D_2^{-1}E_2$, $\bar{E}_2 = E_2$, $\bar{B}_1 = B_1 - D_1D_2^{-1}B_2$, and $\bar{B}_2 = B_2$.

System (10) is then transformed into following sub-systems:

$$\begin{cases} \dot{z}_1(t) = \bar{A}_{11}z_1(t) + \bar{A}_{12}z_2(t) + \bar{\Phi}_1(z,u) + \bar{E}_1f_a(t) \\ \qquad + \bar{B}_1u(t) \\ v_1(t) = C_{11}z_1(t) \end{cases} \tag{11}$$

$$\begin{cases} \dot{z}_2(t) = \bar{A}_{21}z_1(t) + \bar{A}_{22}z_2(t) + \bar{\Phi}_2(z,u) + \bar{E}_2f_a(t) \\ \qquad + \bar{D}_2d(t) + \bar{B}_2u(t) \\ v_2(t) = C_{22}z_2(t) \end{cases} \tag{12}$$

System (8) is converted as follows:

$$\begin{cases} \dot{z}_a(t) = A_sz_a(t) + B_sC_2T^{-1}z(t) + B_sFf_s(t) \\ y_a = z_a(t) \end{cases} \tag{13}$$

where $z = (\, z_1 \ z_2 \,)^T$, $B_s \in R^{m\times m}$, $C_2 \in R^{m\times n}$, and $T^{-1} \in R^{n\times n}$; therefore, $B_sC_2T^{-1} \in R^{m\times n}$. Afterward, we assume that $B_sC_2T^{-1} = (\, A_1 \ A_2 \,)$, where $A_1 \in R^{m\times(n-q)}$ and $A_2 \in R^{m\times q}$. System (13) is converted as follows:

$$\begin{cases} \dot{z}_a(t) = A_sz_a(t) + A_1z_1(t) + A_2z_2(t) + B_sFf_s(t) \\ y_a = z_a(t) \end{cases} \tag{14}$$

Figure 1 illustrates the above series of linear coordinate transformation process.

Three sub-systems, *i.e.*, (11), (12), and (14), are obtained from system (1) by designing first-order low-pass fil-

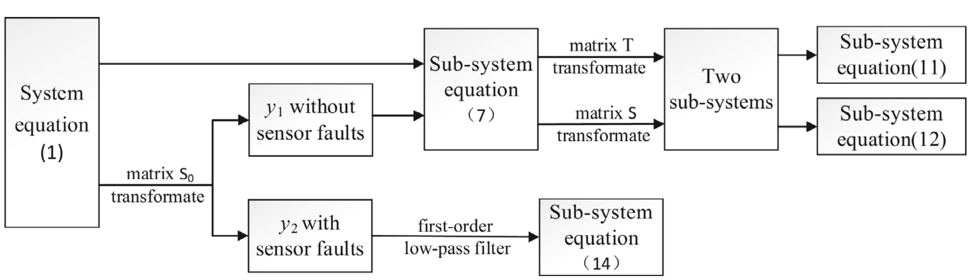

Figure 1. Coordinate transformation diagram.

ter and matrix transformation. Note that the uncertainties only exist in sub-system formulated in (12), meanwhile it has actuator faults while sub-system formulated in (11) contains actuator faults and sub-system formulated in (14) has sensor faults. Corresponding to the above sub-systems, three sliding mode observers are designed, the actuator faults, sensor faults, as well as the unknown input disturbances are reconstructed separately as a result. The design will be introduced in the following sections.

3. Observer Design

Assumption 3. *$(\bar{A}_{11},C_{11})$ and $(\bar{A}_{22},C_{22})$ are observable.*

Assumption 4. *The $\bar{\Phi}_1$ and $\bar{\Phi}_2$ are Lipschtiz terms such that,*

$$\left\|\bar{\Phi}_1(z,u)-\bar{\Phi}_1(\hat{z},u)\right\| \leq \gamma_4\|z-\hat{z}\| \leq \gamma_4\left(\|e_1\|+\|e_2\|\right)$$

$$\left\|\bar{\Phi}_2(z,u)-\bar{\Phi}_2(\hat{z},u)\right\| \leq \gamma_5\|z-\hat{z}\| \leq \gamma_5\left(\|e_1\|+\|e_2\|\right)$$

where γ_4 and γ_5 are known positive Lipschtiz constant.

The sliding mode variable structure observers are designed for the converted systems (11), (12), and (14), respectively:

$$\begin{cases} \dot{\hat{z}}_1(t) = \bar{A}_{11}\hat{z}_1(t) + \bar{A}_{12}\hat{z}_2(t) + \bar{\Phi}_1(\hat{x},u) + \bar{E}_1 r_1(t) \\ \qquad + \bar{B}_1 u(t) + L_1(v_1(t)-\hat{v}_1(t)) \\ \hat{v}_1(t) = C_{11}\hat{z}_1(t) \end{cases} \tag{15}$$

$$\begin{cases} \dot{\hat{z}}_2(t) = \bar{A}_{21}\hat{z}_1(t) + \bar{A}_{22}\hat{z}_2(t) + \bar{\Phi}_2(\hat{x},u) + \bar{E}_2 r_2(t) \\ \qquad + \bar{B}_2 u(t) + L_2(v_2(t)-\hat{v}_2(t)) \\ \hat{v}_2(t) = C_{22}\hat{z}_2(t) \end{cases} \tag{16}$$

$$\begin{cases} \dot{\hat{z}}_a(t) = A_s\hat{z}_a(t) + A_1\hat{z}_1(t) + A_2\hat{z}_2(t) + r_3(t) \\ \hat{y}_a = \hat{z}_a(t) \end{cases} \tag{17}$$

where $r_1(t)$, $r_2(t)$, and $r_3(t)$ represent the input signals of the sliding mode variable structure observers:

$$\begin{aligned} r_1(t) &= \rho_1 sgn(F_1(v_1-\hat{v}_1)) \\ r_2(t) &= \rho_2 sgn(F_2(v_2-\hat{v}_2)) \\ r_3(t) &= \rho_3 sgn(z_a-\hat{z}_a) \end{aligned}$$

where F_1 and F_2 are the gain matrices, and ρ_1, ρ_2, and ρ_3 are positive scalar to be determined.

Assumption 5. *F_1 and F_2 and symmetric positive definite matrices P_1 and P_2 that meet the following equations*:

$$\begin{aligned} P_1\bar{E}_1 &= C_{11}^{\mathrm{T}}F_1^{\mathrm{T}} \\ P_2\bar{E}_2 &= C_{22}^{\mathrm{T}}F_2^{\mathrm{T}} \end{aligned}$$

Assumption 5 is a normal hypothesis in fault isolation [26], [27].

Assumption 3 ensures the existence of the matrices L_1 and L_2 that make A_{01} and A_{02} stable:

$$\begin{aligned} \bar{A}_{11} - L_1\bar{C}_{11} = A_{10} \\ \bar{A}_{22} - L_2\bar{C}_{22} = A_{20} \end{aligned}$$

If the definitions of state estimation errors are $e_1(t) = z_1(t) - \hat{z}_1(t)$, $e_2(t) = z_2(t) - \hat{z}_2(t)$, $e_a(t) = z_a(t) - \hat{z}_a(t)$, and $e = (e_1 \quad e_2)^{\mathrm{T}}$ as well as the output estimation errors are defined as $e_{v1}(t) = v_1(t) - \hat{v}_1(t) = C_{11}e_1(t)$, $e_{v2}(t) = v_2(t) - \hat{v}_2(t) = C_{22}e_2(t)$, and $e_y(t) = e_a(t) = z_a(t) - \hat{z}_a(t)$. State estimation error dynamics can be obtained from (11), (12), and (14)–(17) as follows:

$$\begin{aligned} \dot{e}_1(t) = &\ (\bar{A}_{11} - L_1C_{11})e_1(t) + \bar{A}_{12}e_2(t) + \bar{\Phi}_1(z,u) \\ &-\bar{\Phi}_1(\hat{z},u) + \bar{E}_1 f_a(t) - \bar{E}_1 r_1(t) \end{aligned} \tag{18}$$

$$\begin{aligned} \dot{e}_2(t) = &\ (\bar{A}_{22} - L_2\bar{C}_{22})e_2(t) + \bar{A}_{21}e_1(t) + \bar{\Phi}_2(x,u) \\ &-\bar{\Phi}_2(\hat{x},u) + \bar{E}_2 f_a(t) - \bar{E}_2 r_2(t) + \bar{D}_2 d(t) \end{aligned} \tag{19}$$

$$\dot{e}_a(t) = A_1e_1(t) + A_2e_2(t) + A_se_a(t) + B_sFf_s(t) - r_3(t) \tag{20}$$

Lemma 1. *For the dynamic equation of the error (18) and (19) and Assumption 4, and the following matrix inequality:*

$$\begin{bmatrix} A_{10}^{\mathrm{T}}P_1 + P_1A_{10} + \xi\gamma^2 I_{n-q} & \bar{A}_{21}^{\mathrm{T}}P_2 + P_1\bar{A}_{12} & P_1 & 0 \\ \bar{A}_{12}^{\mathrm{T}}P_1 + P_2\bar{A}_{21} & A_{20}^{\mathrm{T}}P_2 + P_2A_{20} + \xi\gamma^2 I_q & 0 & P_2 \\ P_1 & 0 & -\xi I_{n-q} & 0 \\ 0 & P_2 & 0 & -\xi I_q \end{bmatrix} < 0$$

holds, ρ_1 and ρ_2 satisfy:

$$\begin{aligned} \rho_1 &> \gamma_2 \\ \rho_2 &> \gamma_2 + \frac{\left\|\bar{D}_2\right\|}{\left\|\bar{E}_2\right\|}\gamma_1 \end{aligned}$$

then e_1 and e_2 are asymptotically convergent, scilicet:

$$\lim_{t\to\infty} e_1(t) = 0, \lim_{t\to\infty} e_2(t) = 0$$

where ξ is a positive constant.

Proof. Consider the Lyapunov function as follows:

$$V = e_1^{\mathrm{T}}P_1e_1 + e_2^{\mathrm{T}}P_2e_2 \tag{21}$$

Differentiate V and substitute the error dynamic systems (18) and (19) yields:

$$\begin{aligned} \dot{V} = &\ e_1^{\mathrm{T}}(A_{10}^{\mathrm{T}}P_1 + P_1A_{10})e_1 + 2e_1^{\mathrm{T}}P_1(\bar{A}_{12}e_2 + \bar{\Phi}_1(z,u) \\ &- \bar{\Phi}_1(\hat{z},u) + \bar{E}_1 f_a(t) - \bar{E}_1 r_1(t)) + e_2^T(A_{20}^{\mathrm{T}}P_2 + P_2A_{20})e_2 \\ &+ 2e_2^{\mathrm{T}}P_2(\bar{A}_{21}e_1 + \bar{\Phi}_2(z,u) - \bar{\Phi}_2(\hat{z},u) + \bar{E}_2 f_a(t) \\ &+ \bar{D}_2 d(t) - \bar{E}_2 r_2(t)) \end{aligned} \tag{22}$$

Let,

$$\bar{A}=\begin{bmatrix}\bar{A}_{11}-L_1C_{11} & \bar{A}_{12}\\ \bar{A}_{21} & \bar{A}_{22}-L_2C_{22}\end{bmatrix},\quad P=\begin{bmatrix}P_1 & 0\\ 0 & P_2\end{bmatrix}$$

then,

$$\begin{aligned}\dot{V}&=e^{\mathrm{T}}(\bar{A}^TP+P\bar{A})e+2e^{\mathrm{T}}P(\bar{\Phi}(z,u)-\bar{\Phi}(\hat{z},u))\\&+2e_1^{\mathrm{T}}P_1(\bar{E}_1f_a(t)-\bar{E}_1r_1(t))\\&+2e_2^{\mathrm{T}}P_2(\bar{E}_2f_a(t)+\bar{D}_2d(t)-\bar{E}_2r_2(t))\end{aligned}\tag{23}$$

Since the inequality $2X^{\mathrm{T}}Y\le\frac{1}{\xi}X^{\mathrm{T}}X+\xi Y^{\mathrm{T}}Y$ is true for any scalar $\xi>0$ [28], then,

$$\begin{aligned}&2e^{\mathrm{T}}P(\bar{\Phi}(z,u)-\bar{\Phi}(\hat{z},u))\le\tfrac{1}{\xi}e^{\mathrm{T}}P^2e\\&+\xi\left(\bar{\Phi}(z,u)-\bar{\Phi}(\hat{z},u)\right)^{\mathrm{T}}\left(\bar{\Phi}(z,u)-\bar{\Phi}(\hat{z},u)\right)\\&\le\tfrac{1}{\xi}e^{\mathrm{T}}P^2e+\xi\gamma^2\|e\|^2\end{aligned}\tag{24}$$

holds.

Based on hypothesis 5,

$$\begin{aligned}&2e_1^{\mathrm{T}}P_1\bar{E}_1f_a(t)-2e_1^{\mathrm{T}}P_1\bar{E}_1r_1(t)\le2\|F_1C_{11}\|\\&\|e_1\|(\gamma_2-\rho_1)\le0\end{aligned}\tag{25}$$

$$\begin{aligned}&2e_2^{\mathrm{T}}P_2\bar{E}_2f_a(t)+2e_2^{\mathrm{T}}P_2\bar{D}_2d(t)-2e_2^{\mathrm{T}}P_2\bar{E}_2r_2(t)\\&\le2\|F_2C_{22}\|\|e_2\|(\gamma_2+\frac{\|\bar{D}_2\|}{\|\bar{E}_2\|}\gamma_1-\rho_2)\le0\end{aligned}\tag{26}$$

Equations (24) to (25) are substituted into (22) to gain the following:

$$\dot{V}\le e^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P+P\bar{A}+\frac{1}{\xi}P^2+\xi\gamma^2I_n)e\tag{27}$$

To make $\dot{V}$ negative definite, it must holds that.

$$\bar{A}^{\mathrm{T}}P+P\bar{A}+\frac{1}{\xi}P^2+\xi\gamma^2I_n<0\tag{28}$$

The following matrix inequality:

$$\begin{bmatrix}\bar{A}^{\mathrm{T}}P+P\bar{A}+\xi\gamma^2I_n & P\\ P & -\xi I_n\end{bmatrix}<0\tag{29}$$

holds, such that,

$$\begin{bmatrix}A_{10}^{\mathrm{T}}P_1+P_1A_{10}+\xi\gamma^2I_{n-q} & \bar{A}_{21}^{\mathrm{T}}P_2+P_1\bar{A}_{12} & P_1 & 0\\ \bar{A}_{12}^{\mathrm{T}}P_1+P_2\bar{A}_{21} & A_{20}^{\mathrm{T}}P_2+P_2A_{20}+\xi\gamma^2I_q & 0 & P_2\\ P_1 & 0 & -\xi I_{n-q} & 0\\ 0 & P_2 & 0 & -\xi I_q\end{bmatrix}<0\tag{30}$$

This makes $e(t)$ globally asymptotically stable, which will converge to zero.

$$\lim_{t\to\infty}e_i(t)=0,\quad i=1,2\tag{31}$$

Remark. *Lemma 1 implies that e_1 and e_2 are bounded, i.e., t_0 exists, when $t>t_0$*

$$\|e_1\|\le\delta_1,\quad\|e_2\|\le\delta_2\tag{32}$$

where δ_1 and δ_2 are two positive scalars with finite values.

Lemma 2. *Define sliding mode variable structure surfaces $s_1=F_1e_{v1}$, $s_2=F_2e_{v2}$, and $s_3=e_a$. Suppose Assumptions 4 and 5 and inequality (32) hold, meanwhile, if $\rho_i(i=1,2,3)$ is select as follow:*

$$\rho_1\ge\frac{(\|\bar{A}_{11}-L_1C_{11}\|+\gamma_4)\,\delta_1+(\|\bar{A}_{12}\|+\gamma_4)\,\delta_2+\|\bar{E}_1\|\gamma_2+K_1}{\lambda_{\min}\left(\bar{E}_1^TP_1\bar{E}_1\right)},$$

$$\rho_2\ge\frac{\begin{aligned}&\|F_2C_{22}\|((\|\bar{A}_{22}-L_2C_{22}\|+\gamma_5)\,\delta_2+(\|\bar{A}_{21}\|+\gamma_5)\,\delta_1\\&+\|\bar{E}_2\|\gamma_2+\|D_2\|\gamma_1)+K_2\end{aligned}}{\lambda_{\min}\left(\bar{E}_2^TP_2\bar{E}_2\right)},$$

$$\rho_3>\|A_1\|\delta_1+\|A_2\|\delta_2+\|B_sF\|\gamma_5$$

Then dynamic equation of the error (18)–(20) will be driven to the corresponding $s_i(i=1,2,3)$ in finite time. Where K_1 and K_2 are positive constants.

Proof. **(1)** For system (18), consider a Lyapunov function:

$$V_1=\frac{1}{2}s_1^{\mathrm{T}}s_1\tag{33}$$

Differentiate the above formula and substitute (18):

$$\begin{aligned}\dot{V}_1&=s_1^TF_1C_{11}((\bar{A}_{11}-L_1C_{11})e_1(t)+\bar{A}_{12}e_2(t)+\bar{\Phi}_1(z,u)\\&-\bar{\Phi}_1(\hat{z},u)+\bar{E}_1f_a(t))-\rho_1s_1^T\bar{E}_1^TP_1\bar{E}_1\frac{s_1}{\|s_1\|}\end{aligned}\tag{34}$$

From (32), it follows that,

$$\begin{aligned}&\dot{V}_1\le\|s_1\|(\|F_1C_{11}\|((\|\bar{A}_{11}-L_1C_{11}\|+\gamma_4)\,\delta_1\\&+(\|\bar{A}_{12}\|+\gamma_4)\,\delta_2+\|\bar{E}_1\|\gamma_2)-\rho_1\lambda_{\min}\left(\bar{E}_1^TP_1\bar{E}_1\right))\end{aligned}\tag{35}$$

ρ_1 is designed such that,

$$\rho_1\ge\frac{\begin{aligned}&(\|\bar{A}_{11}-L_1C_{11}\|+\gamma_4)\,\delta_1+(\|\bar{A}_{12}\|+\gamma_4)\,\delta_2\\&+\|\bar{E}_1\|\gamma_2+K_1\end{aligned}}{\lambda_{\min}\left(\bar{E}_1^TP_1\bar{E}_1\right)}\tag{36}$$

Then,

$$\dot{V}_1\le-K_1\|s_1\|\tag{37}$$

(2) Consider a Lyapunov function candidate:

$$V_2=\frac{1}{2}s_2^{\mathrm{T}}s_2\tag{38}$$

Substitute (19) into the derivative of the above formula:

$$\begin{aligned}&\dot{V}_2=s_2^TF_2C_{22}((\bar{A}_{22}-L_2C_{22})e_2(t)+\bar{A}_{21}e_1(t)\\&+\bar{\Phi}_2(z,u)-\bar{\Phi}_2(\hat{z},u)+\bar{E}_2f_a(t)+D_2d(t))\\&-\rho_2s_2^T\bar{E}_2^TP_2\bar{E}_2\tfrac{s_2}{\|s_2\|}\end{aligned}\tag{39}$$

It follows that,

$$\dot{V}_2 \leq \|s_2\|(\|F_2C_{22}\|((\|\bar{A}_{22}-L_2C_{22}\|+\gamma_5)\,\delta_2+(\|\bar{A}_{21}\|+\gamma_5)\,\delta_1 + \|\bar{E}_2\|\gamma_2+\|D_2\|\gamma_1)-\rho_2\lambda_{\min}\left(\bar{E}_2^TP_2\bar{E}_2\right)) \tag{40}$$

ρ_2 is designed such that,

$$\rho_2 \geq \frac{\|F_2C_{22}\|((\|\bar{A}_{22}-L_2C_{22}\|+\gamma_5)\,\delta_2+(\|\bar{A}_{21}\|+\gamma_5)\,\delta_1 + \|\bar{E}_2\|\gamma_2+\|D_2\|\gamma_1)+K_2}{\lambda_{\min}\left(\bar{E}_2^TP_2\bar{E}_2\right)}$$

then,

$$\dot{V}_2 \leq -K_2\|s_2\| \tag{41}$$

(3) Choose Lyapunov function as:

$$V_3 = \frac{1}{2}s_3^{\mathrm{T}}s_3 \tag{42}$$

Differentiate the above formula and substitute (20):

$$\dot{V}_3 = e_a^{\mathrm{T}}\dot{e}_a = e_a^{\mathrm{T}}(A_1e_1+A_2e_2+A_se_a)+e_a^{\mathrm{T}}B_sFf_s(t)-e_a^{\mathrm{T}}r_3(t) \tag{43}$$

where A_s is a negative definite matrix which will be designed. $-K_3=\lambda_{\max}(A_s)$, where K_3 is a positive constant.

$$\dot{V}_3 \leq \lambda_{\max}(A_s)\|e_a\|^2+(\|A_1\|\delta_1+\|A_2\|\delta_2+\|B_sF\|\gamma_3-\rho_3)\|e_a\|$$

Choose,

$$\rho_3 > \|A_1\|\delta_1+\|A_2\|\delta_2+\|B_sF\|\gamma_3$$

then,

$$\dot{V}_3 \leq -K_3\|e_a\|^2 \leq 0 \tag{44}$$

The sliding mode is reached after a finite time [29]. The sliding manifold is defined by $s_i=\dot{s}_i=0(i=1,2,3)$

4. Fault Reconstruction and Disturbance Estimation

Theorem. *Let (1) be the corresponding sliding mode observer for (15)–(17), the fault reconstruction is given by* $f_a(t)=\rho_1\ sgn(F_1e_{v1})$, $f_s(t)=(B_sF)^{-1}\rho_3sgn(e_a)$.

Unknown input disturbances $d(t)=\bar{D}_2^{-1}\bar{E}_2(\rho_2\ sgn(F_2e_{v2})-\rho_1sgn(F_1e_{v1}))$.

Proof. When system arrives at the sliding mode surface, $s_i=\dot{s}_i=0(i=1,2,3)$, then,

$$F_1e_{v1}=F_1\dot{e}_{v1}=F_1C_{11}\dot{e}_1=0 \tag{45}$$

$$F_2e_{v2}=F_2\dot{e}_{v2}=F_2C_{22}\dot{e}_2=0 \tag{46}$$

$$e_a=\dot{e}_a=0 \tag{47}$$

From (45), it follows that,

$$F_1C_{11}((\bar{A}_{11}-L_1C_{11})e_1(t)+\bar{A}_{12}e_2(t)+\bar{\Phi}_1(z,u) - \bar{\Phi}_1(\hat{z},u)+\bar{E}_1f_a(t)-\bar{E}_1r_1(t))=0 \tag{48}$$

From (31), it follows that,

$$f_a(t)\approx r_1(t)=\rho_1sgn(F_1e_{v1}) \tag{49}$$

From (47), the following equation is obtained:

$$A_1e_1(t)+A_2e_2(t)+A_se_a(t)+B_sFf_s(t)-r_3(t)=0 \tag{50}$$

$F\in R^{m\times m}$ is a nonsingular matrix and $B_s\in R^{m\times m}$ is a full rank matrix to be designed. Therefore, a matrix B_s exists to make B_sF to be a nonsingular matrix, and from (31), it follows that,

$$f_s(t)\approx(B_sF)^{-1}r_3(t)=(B_sF)^{-1}\rho_3sgn(e_a) \tag{51}$$

From (46), it follows that,

$$F_2C_{22}((\bar{A}_{22}-L_2\bar{C}_{22})e_2(t)+\bar{A}_{21}e_1(t)+\bar{\Phi}_2(x,u) - \bar{\Phi}_2(\hat{x},u)+\bar{E}_2f_a(t)-\bar{E}_2r_2(t)+\bar{D}_2d(t))=0 \tag{52}$$

According to (31), the following equation is obtained:

$$d(t)\approx\bar{D}_2^{-1}(\bar{E}_2r_2(t)-\bar{E}_2f_a(t))=\bar{D}_2^{-1}\bar{E}_2(\rho_2sgn(F_2e_{v2})-\rho_1sgn(F_1e_{v1})) \tag{53}$$

Decreasing chattering in the sliding mode by using a consecutive function which substitutions the sign function $sgn(s)$ in (15)–(17).

$$sgn(F_1e_{v1}) = \frac{F_1e_{v1}}{|F_1e_{v1}|+\sigma_1},\ sgn(F_2e_{v2}) = \frac{F_2e_{v2}}{|F_2e_{v2}|+\sigma_2},\ sgn(e_a)=\frac{e_a}{|e_a|+\sigma_3}$$

where σ_1, σ_2, and σ_3 are three minor positive constants.

5. Simulation Example

Two examples are given in this section. The first single actuator and sensor fault system is an actual single chain robot. In the second example, a theoretical model is selected to simulate a nonlinear system with two sensor faults and two actuator ones

Example 5.1. The motion equation of a single link manipulator is given by [30], [31]:

$$\begin{cases} J_1\ddot{q}_1+F_n\dot{q}_1+k(q_1-q_2)+mgl\sin q_1=0 \\ J_m\ddot{q}_2+F_m\dot{q}_2-k(q_1-q_2)=u \end{cases} \tag{54}$$

where q_1 and q_2 are the link and rotor displacement, respectively. The robot parameters are include k=2

Nm/rad, F_m=1, F_1=0.5 Nm/(rad/s), J_m=1 Nm^2, J_1=2 Nm^2, m=0.15 kg, g=9.8 m/s^2, and l=0.3 m. The input $u = \sin(5t) + 4\sin(20t)$.

By choosing $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, and $x_4 = \dot{q}_2$. The coefficient matrix corresponding to (1) is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.25 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -2 & -1 \end{bmatrix},$$

$$f(x,u,t) = \begin{bmatrix} 0 \\ -0.2205\sin x_1 \\ 0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 3 \\ 4 \\ 4 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{1}{2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1\;0\;0\;0 \\ 0\;0\;1\;0 \\ 0\;1\;0\;1 \\ 0\;0\;0\;1 \end{bmatrix}, \quad F = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

The actuator fault is estimated as:

$$\hat{f}_a(t) = r_1(t) = \rho_1 sgn(F_1 e_{v1}) = \rho_1 \frac{e_1 + e_3}{|e_1 + e_3| + \sigma_1}$$

The sensor fault is estimated as:

$$\hat{f}_s(t) = (B_s F)^{-1} \rho_3 sgn(e_a) = 0.5(\rho_3 \frac{e_{a1}}{|e_{a1}| + \sigma_3})$$

The disturbance is estimated as:

$$\begin{aligned} \hat{d}(t) &= \bar{D}_2^{-1}\bar{E}_2(\rho_2 sgn(F_2 e_{v2}) - \rho_1 sgn(F_1 e_{v1})) \\ &= \rho_2 \frac{e_4}{|e_4| + \sigma_2} - \rho_1 \frac{e_1 + e_3}{|e_1 + e_3| + \sigma_1} \end{aligned}$$

The observer parameters are chosen as $\rho_1 = 32$, $\rho_2 = 60$, $\rho_3 = 20$, $\sigma_1 = 0.1$, $\sigma_2 = 0.02$, and $\sigma_3 = 0.01$. The simulation of $f_a = 2\sin 40t + 2\sin 5t$ is realized by two superimposed sine waves, which represents incipient fault. The sensor fault $f_s(t)$ is simulated using white noise by joining abrupt and intermittent faults. The amplitude is 3 and the sampling period is 0.02 s. The disturbance $d(t)$ is simulated by a sinusoidal, which is $d(t) = 4\sin 20t$. The incipient value of state variable x is given by [0, 0, -3, $-2]^T$.

Figures 2–5 show four state variable observations. Obviously the observers converge rapidly, which is the basis of fault reconstruction.

Figures 6 and 7 indicate the reconstruction results of actuator fault f_a and disturbance $d(t)$. Figures 8 and 9 show the estimation results of filter state and sensor fault f_s. The aforementioned simulations show that both of the actuator fault and sensor fault can be precisely reconstructed.

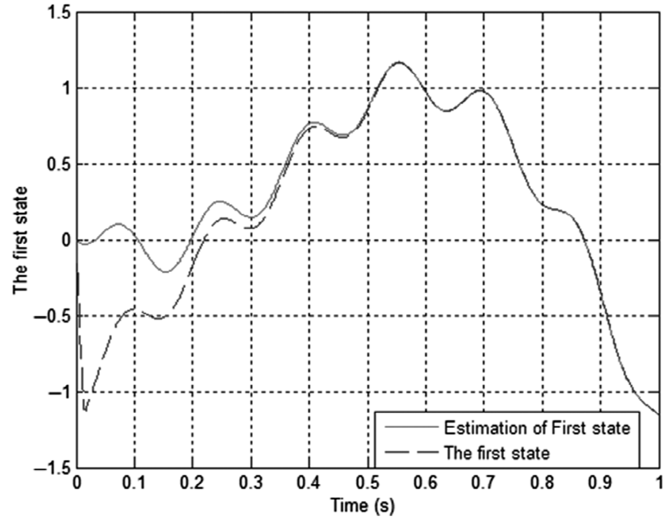

Figure 2. Estimate of function x_1.

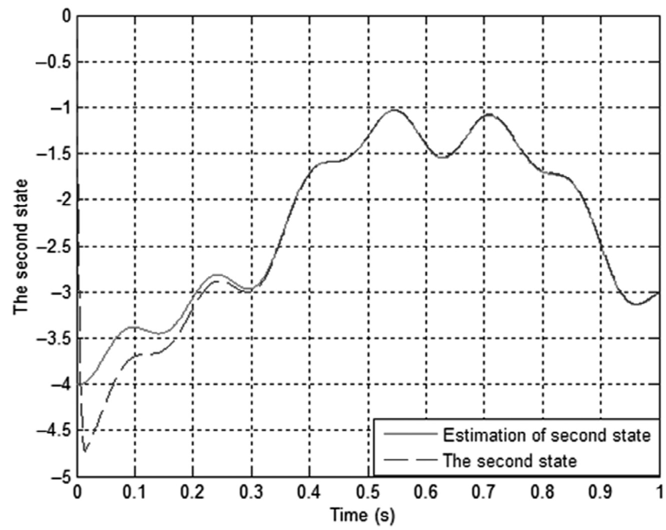

Figure 3. Estimate of function x_2.

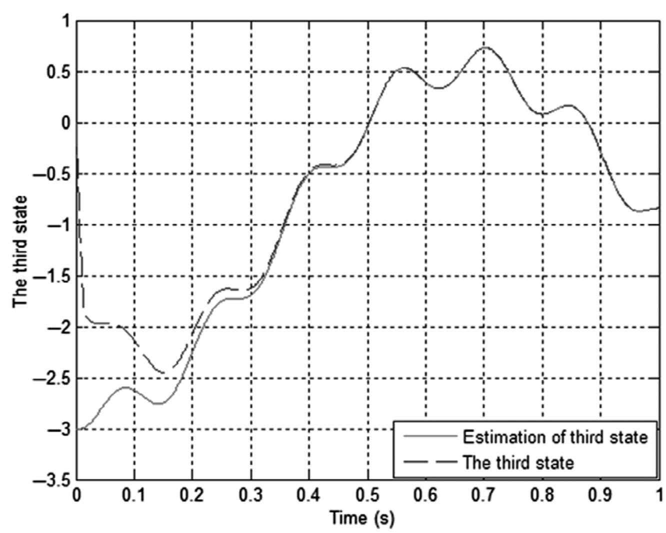

Figure 4. Estimate of function x_3.

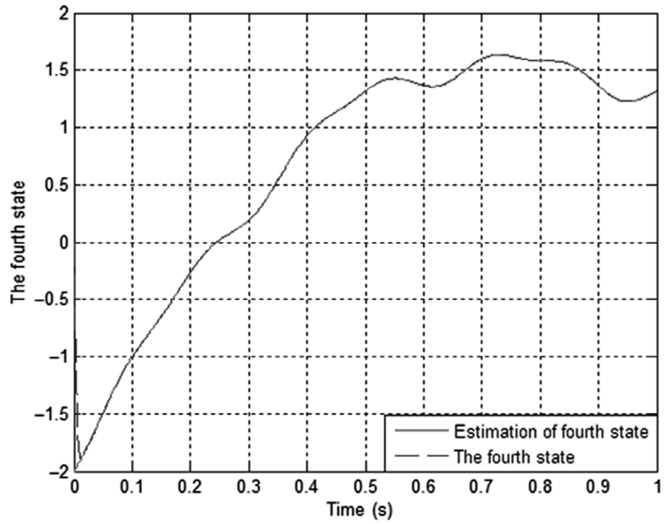

Figure 5. Estimate of function x_4.

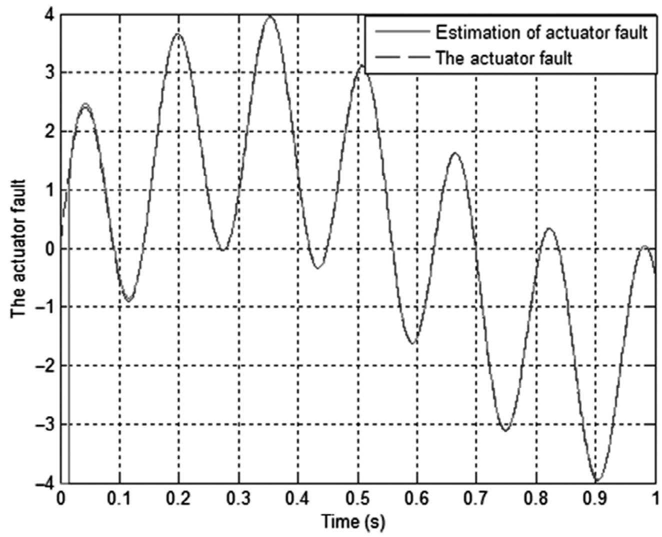

Figure 6. Estimate of function f_a.

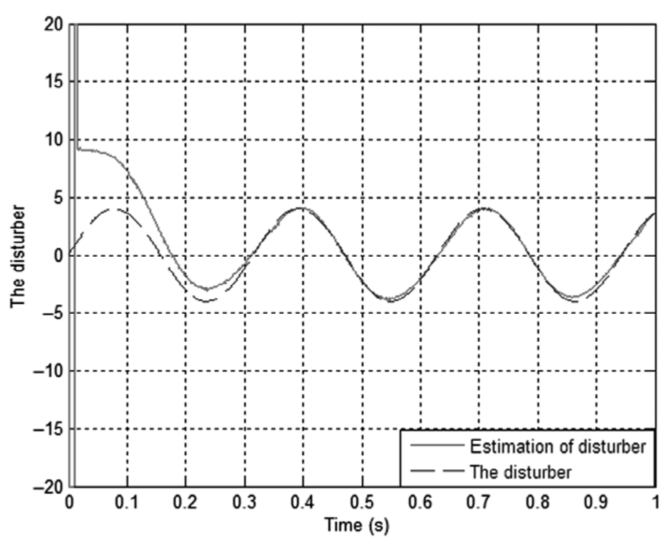

Figure 7. Estimate of function $d(t)$.

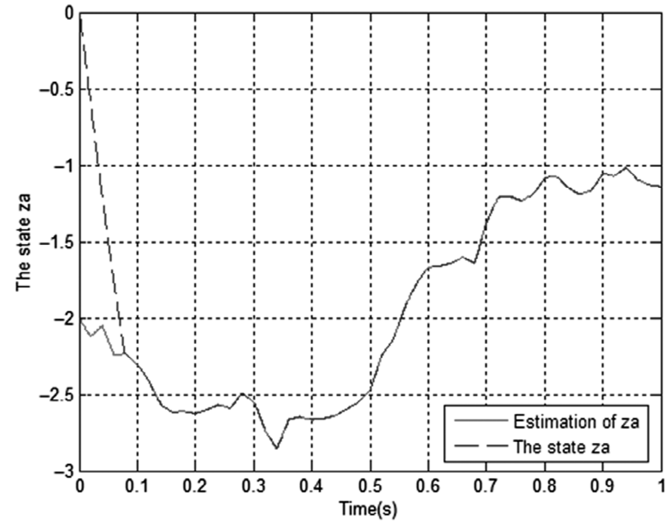

Figure 8. Estimate of function z_a.

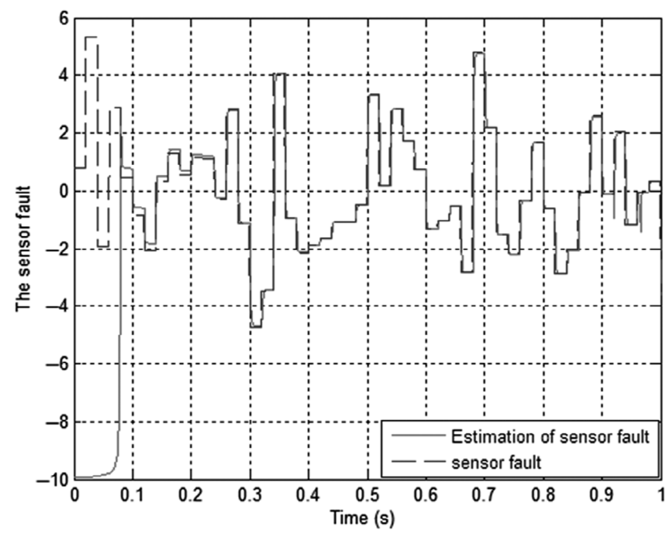

Figure 9. Estimate of function f_s.

Example 5.2. Consider the following system:

$$\begin{cases} \dot{x}(t) = Ax + \Phi(x,u) + Ef_a(t) + Dd(t) + Bu(t) \\ y(t) = Cx(t) + F_s f_s(t) \end{cases} \tag{55}$$

where,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -0.15 & -0.05 & 1.5 & 0 & -1 & -0.03 \\ 0.4 & -1 & -6 & 0 & 0.4 & -1.6 \\ 0.5 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & -25 \end{bmatrix},$$

$$\Phi(x,u) = \begin{bmatrix} \sin x_2 \\ 0 \\ x_1+\sin x_6 \\ 0 \\ 0 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 1 & 0 \\ 0 & 1.5 \\ 6 & 0 \\ 0 & 2 \\ 2 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3 & 0 \\ 0 & 4 \\ 1 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 2 & 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}, F_s = \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$d(t) = \begin{bmatrix} d_1(t) & d_2(t) \end{bmatrix}^{\mathrm{T}}$, $f_a(t) = \begin{bmatrix} f_{a1}(t) & f_{a2}(t) \end{bmatrix}^{\mathrm{T}}$, and $f_s(t) = \begin{bmatrix} f_{s1}(t) & f_{s2}(t) \end{bmatrix}^{\mathrm{T}}$.

From (51), the reconstruction arithmetic of actuator fault $f_a(t)$ is obtained.

$$\hat{f}_{a1}(t) = \rho_{11}\frac{2e_1}{|2e_1|+\sigma_1}$$

$$\hat{f}_{a2}(t) = \rho_{12}\frac{3e_2}{|3e_2|+\sigma_1}$$

From (53), the reconstruction arithmetic of sensor fault $\mathrm{f}_s(\mathrm{t})$ is obtained.

$$\hat{f}_{s1}(t) = 0.5\rho_{31}\frac{e_{a1}}{|e_{a1}|+\sigma_3}$$

$$\hat{f}_{s2}(t) = 0.5\rho_{32}\frac{e_{a2}}{|e_{a2}|+\sigma_3}$$

From (55), the estimation arithmetic of unknown input disturbance is obtained.

$$\hat{d}_1(t) = 2(\rho_{21}\frac{4e_5}{|4e_5|+\sigma_2} - \rho_{11}\frac{2e_1}{|2e_1|+\sigma_1})$$

$$\hat{d}_2(t) = 0.5(\rho_{22}\frac{2e_6}{|2e_6|+\sigma_2} - \rho_{12}\frac{3e_2}{|3e_2|+\sigma_1})$$

The observed parameters are as follows: $\rho_1 = \begin{bmatrix} \rho_{11} & 0 \\ 0 & \rho_{12} \end{bmatrix} = \begin{bmatrix} 20 & 0 \\ 0 & 40 \end{bmatrix}$, $\rho_2 = \begin{bmatrix} \rho_{21} & 0 \\ 0 & \rho_{22} \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 60 \end{bmatrix}$, $\rho_3 = \begin{bmatrix} \rho_{31} & 0 \\ 0 & \rho_{32} \end{bmatrix} = \begin{bmatrix} 40 & 0 \\ 0 & 30 \end{bmatrix}$, and $\sigma_1 = \sigma_2 = \sigma_3 = 0.01$.

The actuator fault $f_a(t)$, where $f_{a1} = 2\sin 40t + 2\sin 5t$, $f_{a2}(t) = 8\sin(20t)$. The sensor fault $f_{s1}(t) = 5\sin(40t)$, $f_{s2}(t)$ is simulated by a white noise with sample period of 0.02 s. The unknown input disturbances $d_1(t) = 4\sin 15t$ and $d_2(t) = 6\sin(20t)$. The initial value of x is -2. The simulation results are as follows:

Figures 10–15 show that the observers converge quickly, which lays the foundation for estimating faults and disturbances.

Figures 16 and 17 represent the reconstruction of actuator faults f_{a1} and f_{a2}. Figures 18 and 19 represent the estimation results of unknown input disturbances $d_1(t)$ and $d_2(t)$. The simulation results show that the method accurately estimates faults and disturbances at the same time.

Figures 20 and 21 show the estimation results of two filter state. In Fig. 22, the incipient sensor fault is simulated by a sine function, and in Fig. 23, the high frequency sensor fault simulated by white noise.

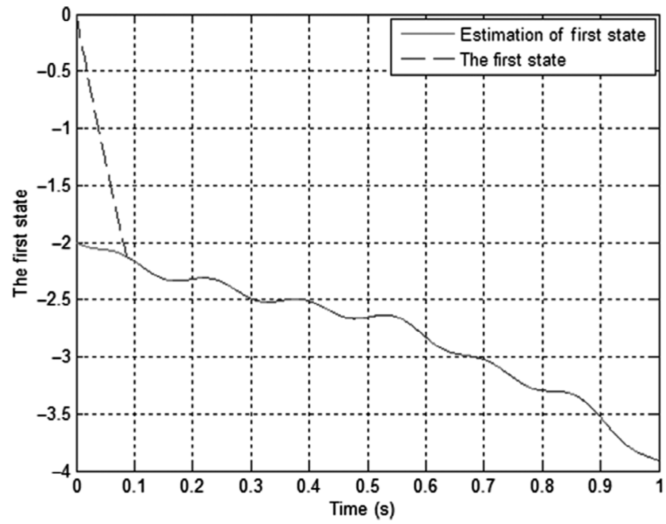

Figure 10. Estimate of function x_1.

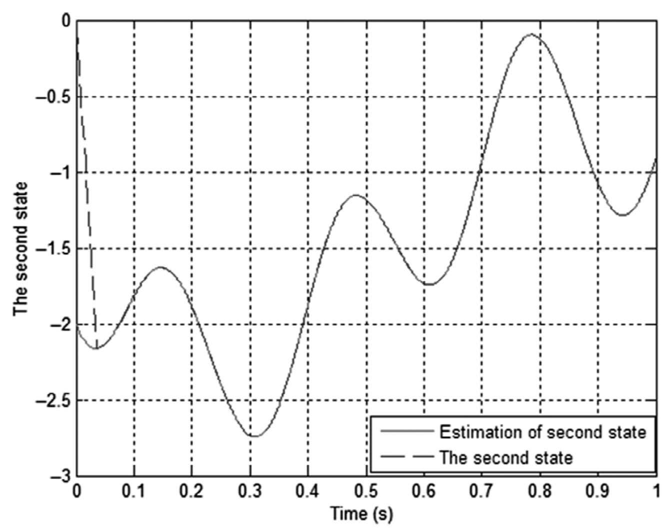

Figure 11. Estimate of function x_2.

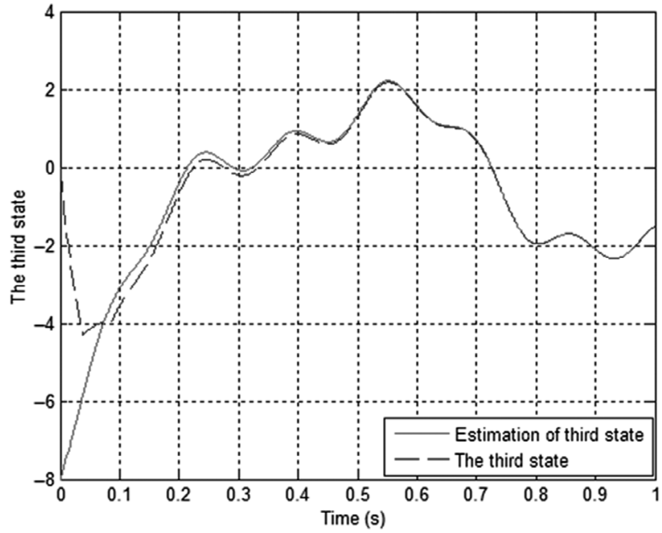

Figure 12. Estimate of function x_3.

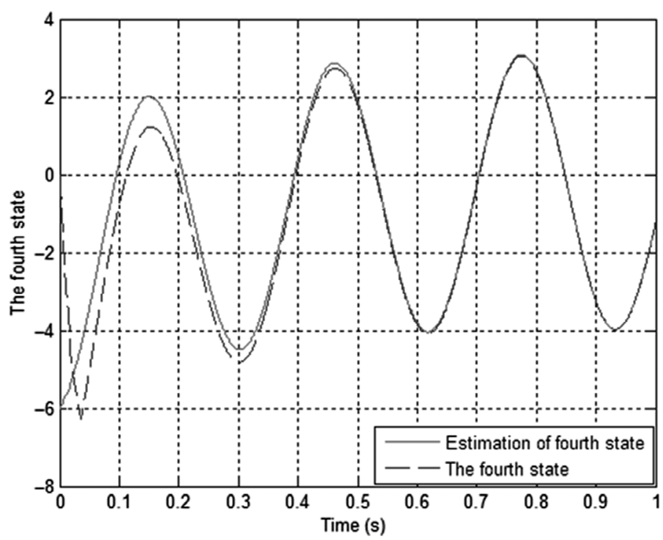

Figure 13. Estimate of function x_4.

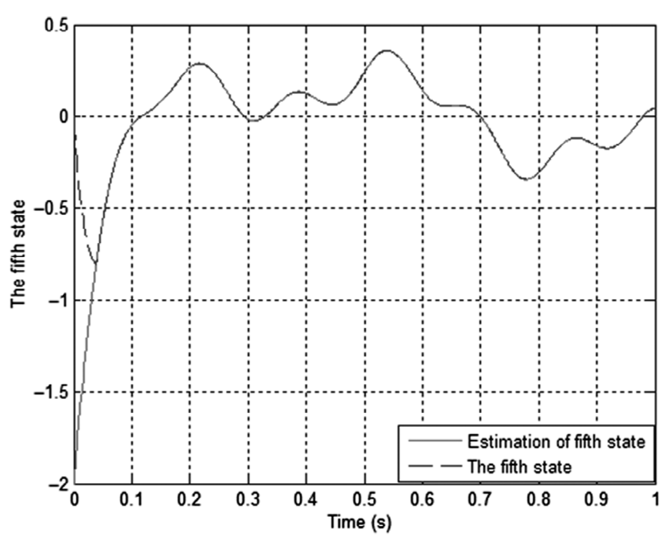

Figure 14. Estimate of function x_5.

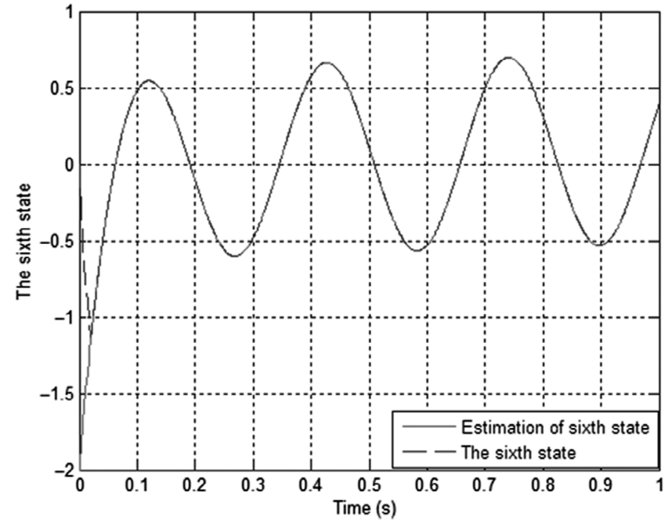

Figure 15. Estimate of function x_6.

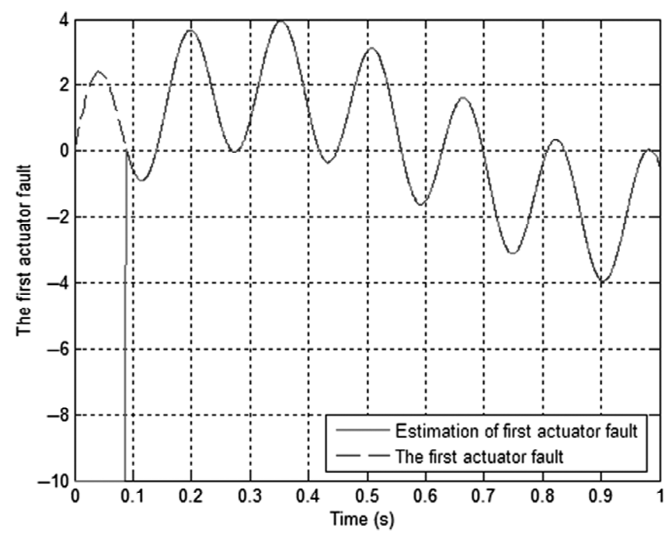

Figure 16. Estimate of function f_{a1}.

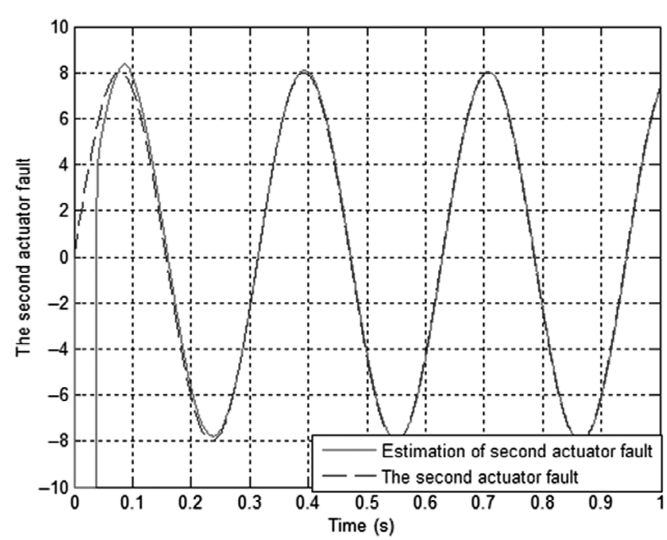

Figure 17. Estimate of function f_{a2}.

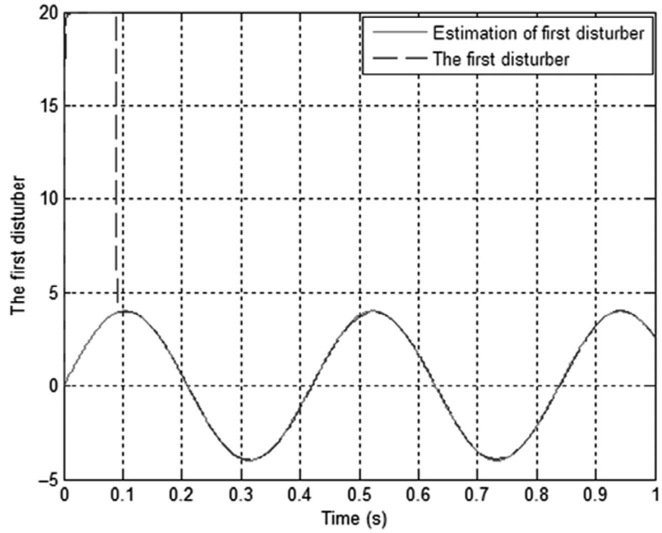

Figure 18. Estimate of function $d_1(t)$.

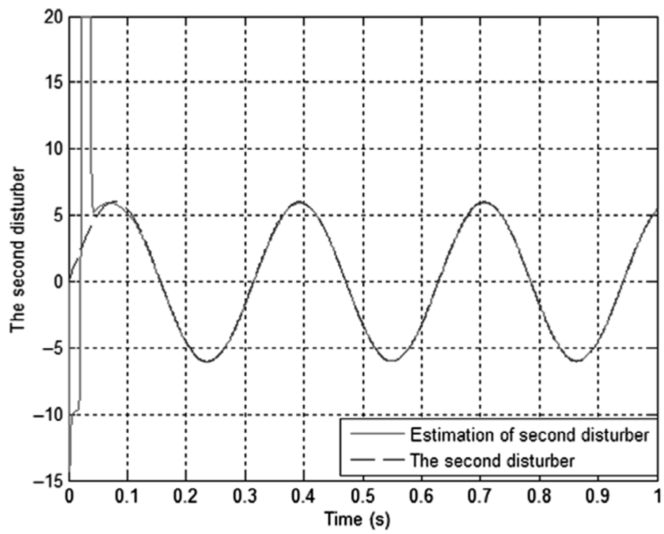

Figure 19. Estimate of function $d_2(t)$.

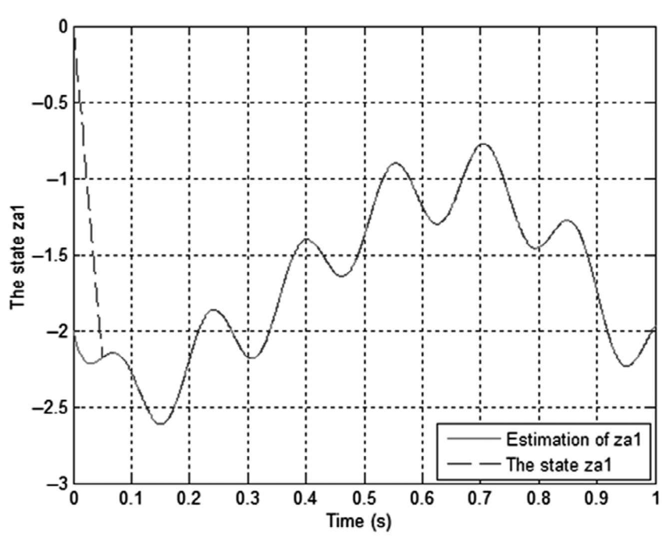

Figure 20. Estimate of function z_{a1}.

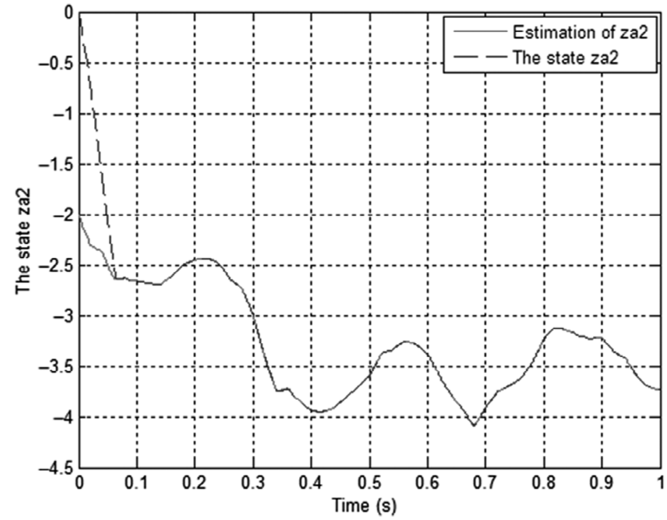

Figure 21. Estimate of function z_{a2}.

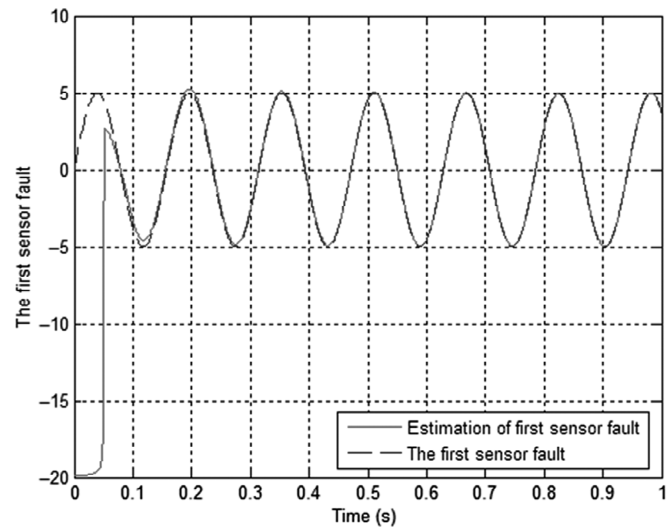

Figure 22. Estimate of function f_{s1}.

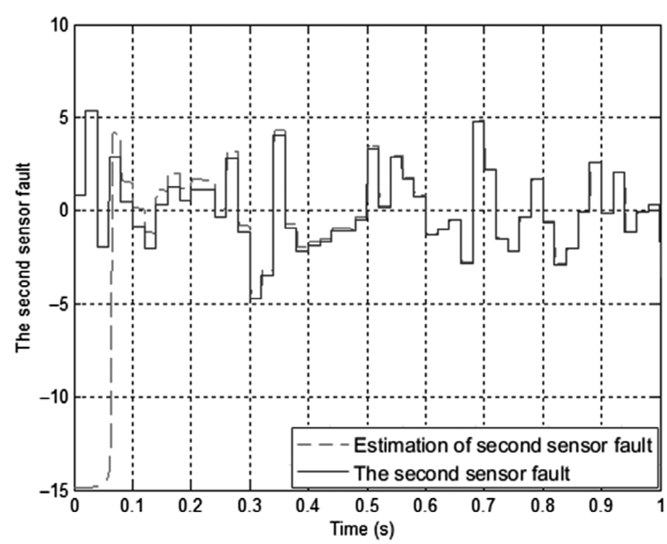

Figure 23. Estimate of function f_{s2}.

6. Conclusion

This paper researched a type of system with actuator faults, sensor faults, and disturbances. A fault reconstruction method for nonlinear system is presented. First, a primary transformation is made on the output equation of the system to obtain two output sub-systems. A filter is designed for the output equation, which contains sensor fault, to convert a sensor fault to an actuator one. The system is transformed to three sub-systems. The accurate reconstruction of actuator and sensor fault, and the estimation of disturbance are both achieved by the designed sliding mode variable structure observer. The applications of fault reconstruction in a robot system and in a six-order nonlinear model with disturbances, respectively, have demonstrated the effectiveness of the presented method. However, an actual industrial system is often more complicated. Therefore, reducing the Lipschtiz condition limit is of great practical meaning for the actual industrial system, and will be carried out in the future.

Acknowledgement

This work was supported by the Natural Science Foundation of China (nos. 61733004, U1934219, and 61773159) and Project of Hunan Provincial Department of Education (no. 19A137).

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